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ABSTRACT

A set of ten teacher-prepared Learning Activity Packages (LAPs) for individualized instruction in topics in pre-algebra, the units cover the decimal numeration system; number theory; fractions and decimals; ratio, proportion, and percent; sets; properties of operations; rational numbers; real numbers; open expressions; and open rational expressions. Each unit contains a rationale for the material; a list of behavioral objectives for the unit; a list of resources including texts (specifying reading assignments and problem sets), tape recordings, and commercial games to be used; a problem set for student self-evaluation; suggestions for advanced study; and references. For other documents in this series, see SE 015 193, SE 015 195, SE 015 196, and SE 015 197. (DT)

ED 069502

U.S. DEPARTMENT OF HEALTH
EDUCATION & WELFARE
OFFICE OF EDUCATION

LEARNING ACTIVITY PACKAGE

DECIMAL NUMERATION SYSTEM

Ninety Six High School
Ninety Six, S.C.

Pre-Algebra

VIEWED BY

J.A.R.

LAP NUMBER 1

WRITTEN BY Diane Evans

FILMED FROM BEST AVAILABLE COPY

RATIONALE

Probably the greatest achievement of man is the development of symbols to communicate facts and ideas. Symbols known as letters developed into the words of many languages of the people of the world. The early Egyptians used a type of picture-writing called "hieroglyphics" for words and numbers. The Babylonians of ancient times made number symbols by pressing the point of a wedge-shaped stick into soft clay tablets. The Roman numeral system was widely used for nearly 2000 years and can still be seen as dates on buildings and chapter numbers in books.

The symbols used in our modern civilized world began in India and were used and passed on to us by the Arabs. Thus, they are called the Hindu-Arabic numerals. Our numbers may also be called decimal numerals because it has ten symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and decimal means ten. In this LAP we will review some of the basic ideas of our decimal numeration system.

BEHAVIORAL OBJECTIVES

At the completion of your prescribed course of study, you will be able to:

1. Given any decimal number written in words, rewrite it as a numeral.
2. Given any decimal numeral, rewrite its name in words.
3. Given any decimal numeral, determine the place value of any given number.
4. Given any product and/or multiple of tens, rewrite it as a power of ten.
5. Given any decimal numeral, rewrite it in expanded form showing the value of each digit using powers of ten.
6. Given an expansion by powers of ten, write each as a decimal numeral.
7. Given two or more decimal numerals, compute their sum, difference, product, and/or quotient.

RESOURCES

Obj. 1

Nichols, read p. 67, Ex. 3 page 67; 15 page 74.
Wollensak tape C-3432: The Decimal System
Wollensak tape C-3431: Ancient Number Systems

Obj. 2

Nichols, read p. 67, Ex. 4, 5 page 68; 16 page 74.

Obj. 3

Nichols, read page 68, Ex. 1-8 pages 6869; 1-5 page 72; 17 page 74.
Wollensak tape C-3301: Understanding Decimals

Obj. 4

Nichols, read page 69, Ex. 1 page 69; 4 page 70; 14 page 74.

Obj. 5

Nichols, read pages 69-70, Ex. 3 page 70.
Wilcox, read pp. 224-225, Ex. 1-10 page 225.

Obj. 6

Nichols, read pages 69-70, Ex. 5 page 71.
Wilcox, read pages 224-225, Ex. 17-22 page 226.

Obj. 7

Nichols, read pp. 69-70, Ex. 1, 3, 6, 8 page 71; 1, 3, 5, 6 page 76; 1,5,7,9 page 84; 5,6,7,8 page 86.

SELF-EVALUATION

Obj.

1 I. Rewrite each of the following as a numeral.

_____ 1. four hundred eighty-seven

_____ 2. ninety-six

_____ 3. two hundred thirty-seven thousand, eighty-one

_____ 4. three thousand, two hundred eighteen

_____ 5. twenty-two thousand, twenty-two

2 II. Rewrite each of the following in words.

6. 21 _____

7. 486 _____

8. 3,410 _____

9. 23,461 _____

10. 104,003 _____

3 III. Write the place value of the underlined digit in each of the following numerals.

11. 6$\underline{8}$4 _____

12. 2$\underline{4}$,687 _____

13. 1$\underline{2}$4,391 _____

14. 222,$\underline{2}$22 _____

15. $\underline{1}$28,821 _____

4 IV. Rewrite each of the following as a power of ten.

_____ 16. 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10

_____ 17. 10,000

_____ 18. 1,000,000,000

_____ 19. 10

_____ 20. 10 x 10 x 10 x 10

_____ 21. 100,000

5 V. Expand the following by powers of ten.

22. 81 _____

23. 20,413 _____

24. 104 _____

25. 231,418 _____

6 VI. Given the following expansions, rewrite each as a decimal numeral.

_____ 26. $2 \times 10^3 + 4 \times 10^2 + 3 \times 10^1 + 7 \times 10^0$

_____ 27. 7×10^8

_____ 28. $8 \times 10^2 + 6 \times 10^1 + 8 \times 10^0$

_____ 29. $7 \times 10^5 + 3 \times 10^4 + 0 \times 10^2 + 5 \times 10^1 + 3 \times 10^0$

7 VII. Perform the following operations.

ADD: 30) $\begin{array}{r} 36.76 \\ 9.46 \\ \hline \end{array}$ 31) 2.431 + 6.82 32) $\begin{array}{r} 8.41 \\ .24 \\ 3.81 \\ 15.62 \\ \hline \end{array}$

SUBTRACT: 33) $\begin{array}{r} 3.6 \\ 1.9 \\ \hline \end{array}$ 34) $\begin{array}{r} 4.62 \\ 3.79 \\ \hline \end{array}$ 35) $\begin{array}{r} .31 \\ .28 \\ \hline \end{array}$

MULTIPLY: 36) $\begin{array}{r} 3.6 \\ 7.9 \\ \hline \end{array}$ 37) $\begin{array}{r} 101.9 \\ .27 \\ \hline \end{array}$ 38) $\begin{array}{r} 2.83 \\ .4 \\ \hline \end{array}$

DIVIDE: 39) $.2\overline{)365.9}$ 40) $.06\overline{)9.6}$ 41) $.001\overline{)50}$

If you have satisfactorily completed your work, take the LAP Test. Consult your teacher first.

ADVANCED STUDY

1. Study the Egyptian numeration system. Give each symbol and its value, then write Egyptian numerals for (1) 35, (2) 387, (3) 1,002,012, and (4) 362,987.
2. Study the Roman numeration system. Give each symbol and its value, then write Roman numerals for (1) 56, (2) 387, (3) 1492, (4) 2348.
3. Write a paper of at least 500 words on the origin of the Hindu-Arabic numeration system.
4. Study either the Egyptian, Babylonian, or Roman systems and answer the following questions in comparing the system to ours.
    1. How does the idea of placement differ from our system?
    2. There was no need for the symbol zero in the ancient systems. Explain why a zero is necessary to write 205 in our system, whereas the ancient systems did not need a zero to write this number.

REFERENCES

Nichols (abbreviated)

Nichols, Eugene D., Pre-Algebra Mathematics, Holt, Rinehart and Winston, Inc., 1970.

Wilcox (abbreviation)

Wilcox, Marie S., Mathematics A Modern Approach, Addison-Wesley Publishing Company, 1971.

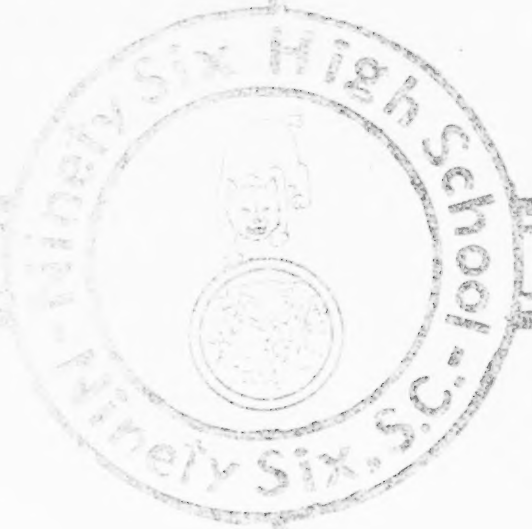
Wollensak Teaching Tapes C-3431: Ancient Number Systems
C-3432: The Decimal System
C-3301: Understanding Decimals

LEARNING

ACTIVITY

PACKAGE

NUMBER THEORY



Pre-Algebra

'IEWED BY

LAP NUMBER 2

WRITTEN BY Diane Evans

RATIONALE

In this LAP you will be studying number theory. Actually, you have been dealing with number theory for many years. For example:

-- when you first learned how to count by fives, you were actually learning how to name the multiples of five.

-- when you computed the lowest common denominator while adding and subtracting fractions, you were actually computing the least common multiple of the denominators.

-- when you identified fractions which were in "simplest" form, you were using the concept of relatively prime numbers.

Not only will you find number theory interesting and fun, but from the above examples, it should be obvioua thay you will find it useful in your future work with fractions.

SECTION 1

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

1. Write and/or identify the definition of factor.
2. List the set of factors (divisors) of a given natural number.
3. Determine if a given number is even or odd.
4. Given any natural number, determine if it is divisible by 2, 3, 5, 9, 10 by using the proper dibisibility test.
5. Given any natural number, classify it as:
   a) a prime number
   b) a composite number
   c) neither prime nor composite
6. Write the prime factorization for any given composite natural number.

RESOURCES 1

Obj. 1

Nichols, read p. 8, Ex. Define factor.

Obj. 2

Nichols, read page 3, Ex. 5 page 3; 8 page 9.
Wilcox, read page 65, Ex. 1-20 even page 65.

Obj. 3

Nichols, read page 8, Ex. 9 page 9.

Obj. 4

Nichols, read pages 1, 6, 7, Ex. 1-8 page 1; 1-3 page 6; 1-6 pages 7-8; 10, 11 page 9.
* Appendix I

Obj. 5

Nichols, read page 8, Ex. 1-5 page 8.
Wollensak Tape C-3010: Prime Numbers

Obj. 6

Nichols, read page 10, Ex. 1-3, 4 b, e, h, 5 every other letter page 10.
Wilcox, read page 66, Ex. 1-20 even page 66.
Wollensak Tape C-3011: Complete Factorization

* required - turn in to teacher

SELF-EVALUATION 1

Obj.
.,3,5,8 I. Define the following terms.

1. factor -

2. even number -

3. odd number -

4. prime number -

5. composite number -

6. relatively prime numbers -

2 II. List the set of factors of the following.

(7) 60 _____

(8) 36 _____

(9) 45 _____

3 III. Determine if each of the following numbers is even or odd. Write E or O in the blank.

____ 10) 687

____ 11) 294

____ 12) 35

____ 13) 42

4 IV. Answer the following.

14. A number is divisible by 2 when it ends in a(an) _____ number.

15. A number is divisible by 3 when the _____ of its digits are divisible by _____.

16. If a number ends in 5 or 0 it is always divisible by _____.

17. If the sum of its digits are divisible by 9 then the number is _____ by _____ .

18. A number that is divisible by 10 must end in _____.

19. Which of the following numbers are divisible by: (May be more than one answer)

_____ a. 2	1. 150
_____ b. 3	2. 48
_____ c. 5	3. 21
_____ d. 9	4. 38
_____ e. 10	5. 909
	6. 45

Choose the best answer.

_____ 20. 7530	a. divisible by 3, 5, and 9
_____ 21. 567	b. divisible by 2, 5, and 10
_____ 22. 452, 765	c. divisible by 3 but not 2 or 5
_____ 23. 116, 112	d. divisible by 5 but not 2 or 9
	e. divisible by 3 and 2 but not 9

5 V. Match.

COLUMN A	COLUMN B
____ 24) 13	A. Prime
____ 25) 1	B. Composite
____ 26) 51	C. neither Prime nor Composite
____ 27) 4	
____ 28) 37	

5 VI. Answer the following.

29. A number which has exactly two different factors is called _____.

30. Which of the following is not prime? _____

a. 17 b. 2 c. 79 d. 51 e. 101

31. The number "2" is the only _____ prime number.

32. Which of the following numbers are composite? _____

a) 25 b) 12 c) 42 d) 29 e) 51

6 VI. 33) Every rational number has only _____ prime factorization.

34) Write prime factorizations for each of the following numbers.

a) 50 _____

b) 24 _____

c) 28 _____

d) 252 _____

e) 720 _____

If you have satisfactorily completed your work, take the Progress Test. Consult your teacher first.

SECTION 2

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

7. Given two or more natural numbers, determine their greatest common factor (GCF).
8. Given two or more natural numbers, determine whether or not they are relatively prime.
9. List or identify at least five multiples of any given natural number.
10. Given two or more natural numbers, determine their best common multiple (LCM).

RESOURCES 2

Obj. 7

Nichols, read page 10, Ex. 6 page 11.
Wilcox, read pages 69-70, Ex. 1-23 even pages 70-71.

Obj. 8

* Appendix II

Obj. 9

Nichols, read page 24, Ex. 1 a, c, e, f page 25

Obj. 10

Nichols, read page 24, Ex. 2 every other letter page 25; 6 page 26.
Wilcox, read page 10, Ex. 1-20 even page 11.

* Required - turn in to teacher.

SELF-EVALUATION 2

Obj.
7 I. Determine the greatest common factor of each of the following pairs of numbers.

_____ 1) 42 and 56

_____ 2) 56 and 96

_____ 3) 54 and 90

_____ 4) 115 and 96

8 II. Answer the following.

_____ 5. By saying "21 and 13 are relatively prime" we mean

a. they are both odd
b. they are both prime
c. their GCF is one
d. they are both composite

_____ 6. Which of the following sequences of numbers are relatively prime?

a. 3 and 51
b. 17 and 92
c. 12 and 28
d. 5, 4, 3

9 III. Give 5 multiples of each of the following numbers.

7) 12 _____

8) 15 _____

9) 8 _____

10) 43 _____

10 IV. 11) List at least 5 multiples of 12 and 8. _____

_____

12) Give the least common multiple of the following sets of numbers.

a. 3 and 5 _____ d. 33 and 22 _____

b. 98 and 80 _____ e. 13 and 20 _____

c. 10 and 15 _____

If you have satisfactorily completed your work, take the LAP Test. Consult your teacher first.

APPENDIX I

Obj. 4

1. Circle the numbers in the following list that are divisible by 2.

   24 1285 3,078 16,479

2. Circle the numbers divisible by 3.

   24 1285 3,078 16,479

3. Circle the numbers divisible by 5.

   989 28,505 41,401 12,465

4. Circle the numbers divisible by 9.

   989 96,307 41,401 12,465

5. Circle the numbers divisible by 10.

   381 970 523 4830

APPENDIX II

Obj. 8

Relatively prime numbers are two or more numbers whose greatest common factor is 1.

Example: 7, 24

Circle the pairs of numbers below which are relatively prime.

1. 4, 20
2. 9, 21
3. 24, 35
4. 16, 40
5. 28, 55
6. 21, 49
7. 36, 55
8. 27, 64
9. 11, 89
10. 7, 37

ADVANCED STUDY

1. Nichols, Exercises 1-9 pages 26-27.
2. Prime factor 10,001.
3. Make a chart showing Erathosthenes' method of determining prime and composite numbers.
4. Write a report explaining the divisibility test for 11.
5. Use the LCM to compute the following.

   (a) $\frac{3}{20} + \frac{5}{16}$

   (b) $\frac{17}{18} + \frac{11}{30} + \frac{22}{35}$

   (c) $\frac{7}{12} + \frac{7}{15}$

   (d) $\frac{5}{9} + \frac{5}{18} + \frac{5}{12}$

REFERENCES

Nichols (abbreviation)

Nichols, Eugene D., Pre-Algebra Mathematics, Holt, Rinehart and Winston, Inc., 1970.

Wilcox (abbreviation)

Wilcox, Marie S., Mathematics A Modern Approach, Addison-Wesley Publishing Company, 1971.

Wollensak Teaching Tapes C-3010: Prime Numbers
C-3011: Complete Factorization

L EARNING

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P ACKAGE

FRACTIONS AND DECIMALS

Ninety Six High School - Ninety Six, S.C. -

Pre-Algebra

EWED BY

LAP NUMBER 3

WRITTEN BY Diane Evans

RATIONALE

In your first Learning Activity Package, you studied how our system of numbers developed over many centuries. As you know by now, mathematics began with the invention of numbers to count by. Prehistoric man's need to count at all was very limited, but as man progressed and became more civilized, he found he had no way to express certain ideas in numbers. For example, if two bananas needed to be divided equally among three children, there was no number to express this. As a result, because of this inexact division, fractions were invented.

In this LAP, you will review the concept of fractions and several related topics. In addition to reviewing the four basic operations with fractions, you will study how fractions and decimals relate.

SECTION 1

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

1. Given any fraction, write it in simplest form.
2. Given two fractions, determine if the first fraction is >, < , or = to the second fraction.
3. Given any list of numerals, select those which name the number one.
4. Given any pair of fractions, compute their:
   a. sum
   b. difference
   c. product
   d. quotient

RESOURCES

Obj. 1

* Appendix I

Obj. 2

Nichols, read pages 106-107, Ex. 4-9, 20-28 pages 107-108.

Obj. 3

Nichols, read page 110, Ex. 1-18 page 110.

* Appendix II

Obj. 4

A. Wilcox, read pages 47-49, Ex. 1-14 even page 47; 1-15 even page 49.

B. Wilcox, read page 89, Ex. 2, 4, 8, 11, 15, 17, 19 pages 89-90.

C. Wilcox, read pages 52-53, Ex. 1-28 even pages 53-54.

D. Wilcox, read page 56, Ex. 1-24 odd page 56.

* required (to be turned to teacher)

SELF-EVALUATION 1

OBJ.

1 I. Write each of the following in simplest form.

_____ (1) $\frac{8}{28}$ _____ (5) $\frac{15}{9}$

_____ (2) $\frac{36}{56}$ _____ (6) $\frac{72}{81}$

_____ (3) $\frac{40}{110}$ _____ (7) $\frac{22}{38}$

_____ (4) $\frac{6}{51}$ _____ (8) $\frac{66}{80}$

2 II. In each blank below, write >, <, or = to make each statement true.

(9) $\frac{4}{7}$ ___ $\frac{9}{12}$ (13) $\frac{36}{56}$ ___ $\frac{9}{14}$

(10) $\frac{15}{30}$ ___ $\frac{5}{6}$ (14) $\frac{15}{16}$ ___ $\frac{4}{5}$

(11) $\frac{2}{3}$ ___ $\frac{4}{7}$ (15) $\frac{7}{8}$ ___ $\frac{4}{5}$

(12) $\frac{4}{9}$ ___ $\frac{24}{54}$ (16) $\frac{140}{4}$ ___ $\frac{35}{1}$

3 III. Circle the following numbers which are names for the number 1.

(17) $\frac{\frac{1}{2}}{\frac{1}{2}}$ (21) $\frac{1}{0}$

(18) $\frac{18}{2 \times 9}$ (22) $\frac{3 + 4 + 1}{2 \times 4}$

(19) $\frac{9 + 1}{2 \times 5}$ (23) $\frac{8 \times 8}{16 \div 4}$

(20) $\frac{6 + 1}{5}$ (24) $\frac{0}{1}$

4 IV. Compute the following:

(25) $4\frac{1}{2} \times 3\frac{2}{9}$ (28) $\frac{2}{3} \div \frac{4}{5}$

(26) $\frac{5}{6} + \frac{3}{5}$ (29) $1\frac{1}{6} \div \frac{2}{3}$

(27) $\frac{5}{3} - \frac{1}{8}$ (30) $\frac{9}{10} \cdot \frac{12}{17}$

4 IV. Compute the following: (cont' from page 3)

(31) $\frac{8}{11} + \frac{4}{5}$

(32) $\frac{33}{6} \cdot \frac{4}{5}$

(33) $\frac{8}{11} \div \frac{6}{22}$

(34) $\frac{4}{5} + \frac{2}{3}$

(35) $3\frac{2}{5} - 1\frac{4}{7}$

(36) $2\frac{2}{3} \times 4\frac{1}{5}$

(37) $\frac{7}{16} - \frac{1}{4}$

(38) $4\frac{3}{4} + 2\frac{3}{5}$

If you have satisfactorily completed your work, take the Progress Test. Consult your teacher first.

SECTION 2

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

5. Name and/or identify the three types of decimals and examples of each.

6. Given any fraction,

   a. Change it to a decimal
   b. Determine if it is repeating or terminating

7. Given any repeating or terminating decimal, change it to a fraction.

8. Given any decimal numeral, round it to the nearest indicated digit.

9. Given any pair of decimal numerals, compute their:

   a. sum
   b. difference
   c. product
   d. quotient

RESOURCES

Obj. 5

* Nichols, read pages 113, 115, and 125, Ex. Appendix III, part I.

Obj. 6

Nichols, read pages 113-114, 115-116, Ex. 1-24 odd page 114; read pages 115-116, Ex. 5-16 even page 116.

* Appendix III, part II.

Obj. 7

Nichols, read page 113, Ex. 37-48 even page 115; read pages 121-122, Ex. 5-19 odd page 122.

Obj. 8

Wilcox, read pages 50, Ex. 1, 3, 7, 9, 11, 13, 18, 21, 24, 28, 31, 33, 36, 37, 39, 40 pages 50-51.

Obj. 9

A. Wilcox, read pages 48-49, Ex. 30-45 even page 49.
* B. Appendix IV
C. Wilcox, read pages 54-55, Ex. 13-36 odd page 55.
D. Wilcox, read pages 57-58, Ex. 21-41 even page 59.

* Required (turn in to teacher)

SELF-EVALUATION 2

OBJ.

5 I. Name the three types of decimals.

(1) _____

(2) _____

(3) _____

6 II. Change the following fractions to decimals and state if each is terminating or repeating.

(4) $\frac{4}{9}$

(5) $\frac{7}{8}$

(6) $\frac{2}{7}$

(7) $\frac{15}{16}$

(8) $\frac{6}{11}$

(9) $\frac{7}{20}$

7 III. Change each decimal below to a fraction and simplify.

(10) .84

(11) $.\overline{43}$

(12) .018

(13) $.\overline{8}$

(14) $.\overline{123}$

(15) .344

8 IV. Round each of the following decimals to the digit indicated.

_____ (16) 2.43 to tenths

_____ (17) 14.873 to hundredths

_____ (18) 286,429 to thousands

_____ (19) 283,469 to hundred thousands

_____ (20) .04329 to thousandths

9 V. Compute the following:

21) 3.416 + 2.983

22) 18.4 - 2.91

23) 3.4 x 2.9

24) .04 x 1.3

25) $.12\overline{)747.6}$

26) $3.9\overline{)4.914}$

27) $.11\overline{)34.54}$

28) $.023\overline{)67.6}$

If you have satisfactorily completed your work, take the LAP Test. Consult your teacher.

APPENDIX I

Obj. 1: Simplify fractions

To simplify a fraction such as $\frac{18}{45}$, we use the Greatest Common Factor. Recall from LAP 2 the greatest common factor is the largest number that can be divided into two or more numbers. The GCF of 18 and 45 is 9.

To simplify or reduce $\frac{18}{45}$, we divide 18 and 45 by 9, thus $\frac{18}{45} = \frac{2}{5}$.

Simplify the following fractions:

1) $\frac{36}{40}$

2) $\frac{25}{80}$

3) $\frac{16}{72}$

4) $\frac{13}{39}$

5) $\frac{27}{35}$

6) $\frac{27}{81}$

7) $\frac{17}{51}$

8) $\frac{55}{88}$

9) $\frac{56}{72}$

10) $\frac{22}{121}$

APPENDIX II

OBJ. 3:

Circle the following numbers which are names for the number one.

1) $\frac{7}{7}$

2) $\frac{2 \times 5}{5 + 5}$

3) $\frac{7 + 8}{2 \times 7}$

4) $\frac{4 + 1}{5 \times 1}$

5) $\frac{\frac{1}{2}}{\frac{1}{2}}$

6) $\frac{4 \times 5}{2 + 10}$

7) $\frac{18 \div 9}{1 + 1}$

8) $\frac{0}{1}$

9) $\frac{1}{1}$

10) $\frac{\frac{1}{4}}{\frac{1}{2}}$

APPENDIX III

Part I: Obj. 5

(a) Name the three types of decimals

1. _____

2. _____

3. _____

(b) For each of the following decimals, state which type of the above decimals it is.

_____ (1) .031

_____ (2) $.8\overline{33}$

_____ (3) .010010001...

_____ (4) 3.472

_____ (5) 2.999

_____ (6) .873873...

_____ (7) $\pi = 3.14159265...$

_____ (8) 3.872594

_____ (9) $.\overline{3}$

_____ (10) .121221222...

PART II: Obj. 6

Change the following fractions to decimals. State if the decimal is repeating or terminating.

_____ 1) $\frac{2}{5}$

_____ 2) $\frac{5}{6}$

_____ 3) $\frac{7}{8}$

_____ 4) $\frac{3}{11}$

_____ 5) $\frac{4}{9}$

_____ 6) $\frac{3}{20}$

APPENDIX IV

OBJ. 9-B: Subtracting decimals

Example: 3.84 - 2.947

Rewrite lining up decimals

$$\begin{array}{r} 3.840 \\ -2.947 \\ \hline .893 \end{array}$$

COMPUTE THE FOLLOWING:

(1) $\begin{array}{r} 23.4 \\ -\ 2.97 \\ \hline \end{array}$

(2) 18.4 - 3.94

(3) 284.7 - 3.997

(4) $\begin{array}{r} 3.49 \\ -1.476 \\ \hline \end{array}$

(5) 48.7 - 2.83

(6) .81 - .04

(7) $\begin{array}{r} 7.62 \\ -.43 \\ \hline \end{array}$

(8) $\begin{array}{r} 19.1 \\ -\ 4.682 \\ \hline \end{array}$

(9) 17.8 - .394

ADVANCED STUDY

Compute any five of the following:

(1) $\frac{3}{4} + \frac{7}{8} - \frac{6}{5}$

(2) $\frac{2}{3} \times \frac{3}{5} + \frac{6}{11}$

(3) $\frac{7}{9} \div \frac{2}{5} = \frac{8}{9}$

(4) $\dfrac{\frac{1}{2} + \frac{3}{4}}{\frac{5}{6} - \frac{1}{3}}$

(5) $\dfrac{\frac{4}{7} - \frac{2}{3}}{\frac{3}{4}}$

(6) $\dfrac{\frac{1}{4} + \frac{1}{3} + \frac{1}{8}}{\frac{1}{2}}$

(7) $(2.4 \times 4.7) \div .02$

(8) $(18.6 + 3.84) - 19.51$

(9) $(.1881 \div .09) + 3.9$

(10) $(19.3 + 2.81) \times .001$

REFERENCES

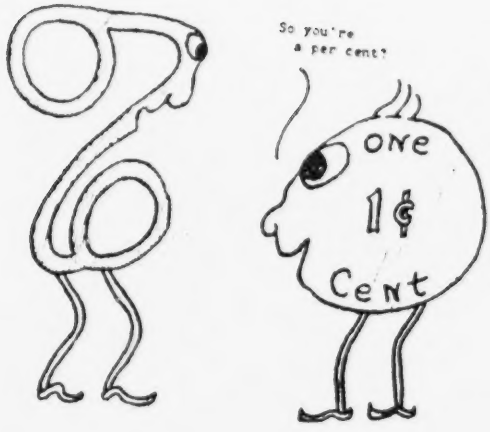
Nichols (abbreviated)

Nichols, Eugene D., Pre-Algebra Mathematics, Holt, Rinehart and Winston, Inc., 1970.

Wilcox (abbreviation)

Wilcox, Marie S., Mathematics A Modern Approach, Addison-Wesley Publishing Company, 1971.

LEARNING
ACTIVITY
PACKAGE



RATIO, PROPORTION, AND PER CENT

Ninety Six High School
Ninety Six, S.C.

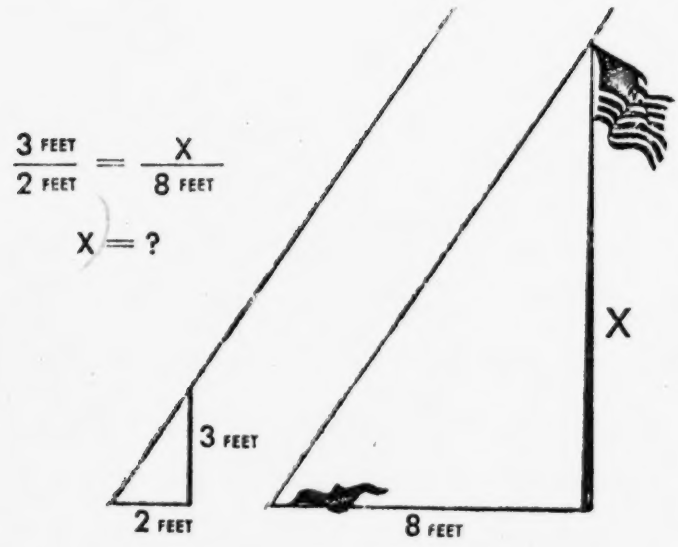
Pre-Algebra

IEWED BY _____

LAP NUMBER 4

WRITTEN BY Diane Evans

RATIONALE



Almost any mathematical problem which you find in everyday living can be solved by means of a proportion. The problem above is one example.

Proportion, decimals, and per cent go hand-in-hand. Banks charge and give interest in per cent. Our monetary system (the money we use) uses decimals, and the whole metric system is based on decimals.

In this LAP you will study these topics in some detail. You will find this knowledge very useful in your everyday life.

In addition to per cent, decimals, and proportion, you will study a little of how they are used in business by studying discounts and commission.

SECTION I

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

1. Given any pair of numbers, write the ratio of the first to the second in lowest terms.
2. Given any pair of ratios, determine whether or not they are proportional.
3. Given any proportion involving a variable, find the number represented by the variable.
4. Given any word problem, construct a proportion and find the correct solution.

* QUIZ

5. Given any fraction, rename it in any of the forms listed below and/or rename each form in any of the other forms:

   a) fraction in simplest form, or whole number if possible.
   b) fraction with 100 as a denominator
   c) decimal
   d) per cent

6. Given sufficient information, calculate

   a) the per cent of a number
   b) commission
   c) discount

* When you complete objectives 1-4, take the short quiz. SEE THE TEACHER.

RESOURCES I

All exercises are required in this LAP. Tapes are optional.

Objective 1

Wilcox, read p. 175, Ex. 1-52 every third number, p. 176.

Objective 2

Nichols, read pp. 166-167, Ex. 3-18 even p. 167.

Objective 3

Wilcox, read pp. 178-180, Ex. 1-20 odd p. 180.

Objective 4

Wilcox, read p. 181, Ex. 1-8 page 181.

QUIZ - SEE TEACHER

Objective 5

* Appendix I
Fraction with 100 as denominator: Nichols Ex. 13-24 page 164.
Fraction to decimal: Wilcox, read 41-42, Ex. 1-20 even p. 42.
Decimal to fraction: Wilcox, Ex. 26-50 odd p. 43.
Fraction to per cent: Wilcox, Ex. 1-20 even p. 44; 1-15 odd p. 46.
Decimal to per cent: Nichols, Ex. 25-31 p. 165.
Per cent to decimal: Nichols, Ex. 1-20 odd p. 163.
Per cent to fraction: Nichols, Ex. 49-56 p. 163.

Wollensak Teaching Tapes:
C-3151 Introduction to Per Cent
C-3152 Changing Fractions to Per Cent
C-3155 Changing Per Cent to Fractions
C-3154 Changing Per Cent to Decimals
C-3157 Changing Decimals to Per Cent

Objective 6

* Appendix II and Wilcox read p. 213, Ex. 1-3 p. 214
Nichols, read p. 179, Ex. 1-4 p. 180.
Nichols, read p. 177, Ex. 1-4 p. 177.

* required - turn in to teacher.

SELF-EVALUATION I

OBJ.

1 I. Find the ratio in lowest terms.

_____ (a) 49 to 14

_____ (b) 15 to 45

_____ (c) $\frac{3}{5} : 1\frac{2}{5}$

_____ (d) $\frac{21}{28}$

_____ (e) $\frac{5}{6}$ to $\frac{7}{8}$

2 II. Which of the following pairs of ratios are proportional?

___ (a) $\frac{3}{5}, \frac{21}{35}$

___ (b) $\frac{42}{28}, \frac{24}{20}$

___ (c) $\frac{68}{17}, \frac{76}{19}$

___ (d) $\frac{103}{49}, \frac{114}{52}$

___ (e) $\frac{3}{5}, \frac{6}{30}$

3 III. Find x in the following sentences so that the two ratios will be proportional.

___ (a) $\frac{4}{36} = \frac{8}{x}$ ___ (c) $\frac{81}{108} = \frac{x}{12}$ ___ (e) $\frac{1\frac{1}{2}}{5} = \frac{2}{x}$

___ (b) $\frac{x}{7} = \frac{12}{3}$ ___ (d) $\frac{45}{3} = \frac{x}{100}$

4 IV. Construct a proportion for each of the following and find the solution.

A. What is the cost of 20 candy bars at 4 for 15¢?

B. A farmer raises 400 bushels of corn on a 20 acre plot. How many bushels can he expect to raise on a 35 acre plot?

(c) A recipe for chocolate chip cookies calls for 2 cups of chocolate chips to make 40 cookies, about how many cups of chips are needed to make 110 cookies?

(d) Mr. Jones uses 7 gallons of gasoline to travel 119 miles. How many gallons are needed to travel 340 miles?

SEE THE TEACHER FOR QUIZ AFTER OBJECTIVE 4.

5 V. Complete the following chart.

Fraction Simplest Form	Hundred as Denominator	Decimal	Per Cent
(a) $\frac{3}{4}$			
(b)	$\frac{60}{100}$		
(c)		.30	
(d)			80%
(e)	$\frac{31}{100}$		
(f)		.375	
(g) $\frac{9}{5}$			
(h)			120%

5 VI. (a) Express as per cents:

(1) $\frac{5}{8}$ (2) .475 (3) $\frac{1}{5}$

(b) Express as decimals:

(1) 38% (2) $\frac{1}{125}$ (3) $\frac{3}{16}$

(c) Express as fractions:

(1) 32% (2) .45 (3) 120%

6 VII. Work the following:

(a) A real estate agent sells a house for $18,600. If his rate of commission is 5%, how much does he receive for the sale of the house?

(b) Jane bought a dress on sale at 20% off. The price was marked $30.00. What was the discount?

(c) There are 150 students in a ninth grade. If 20% of the students take algebra, how many students take algebra?

(d) Northwest High School football team won 80% of their games. If they played 15 games, how many games did they win?

IF YOU HAVE SATISFACTORILY COMPLETED YOUR WORK, TAKE THE LAP TEST. CONSULT YOUR TEACHER FIRST.

APPENDIX I (obj. 5)

Simplify the following:

1) $\frac{8}{32}$

2) $\frac{40}{5}$

3) $\frac{12}{28}$

4) $\frac{36}{27}$

5) $\frac{36}{24}$

6) $\frac{50}{75}$

7) $\frac{10}{30}$

8) $\frac{40}{45}$

9) $\frac{44}{66}$

APPENDIX II (Obj. 6)

I. Find the following per cents of the given numbers.

Example:

40% of 20 $\quad \frac{40}{100} \times \frac{20}{1} = \frac{800}{100} = 8$

1) 10% of 30

2) 9% of 500

3) 150% of 90

4) 15% of 30

5) 30% of 24

6) 120% of 40

7) 80% of 65

8) 2% of 45

II. Complete the following chart.

Fraction Simplest Form	Hundred as Denominator	Decimal	Per Cent
a) $\frac{3}{10}$			
b)		.10	
c)			90%
d) $\frac{1}{8}$			
e)	$\frac{300}{100}$		
f)		.375	
g)	$\frac{62}{100}$		
h)			150%
i) $\frac{9}{5}$			
j)	$\frac{70}{100}$		
k) $\frac{4}{5}$			
l)		$.33\frac{1}{3}$	
m)			$66\frac{2}{3}\%$

ADVANCED STUDY

I. Work the following.

INTEREST

If you need to borrow money, you might go to a bank. Suppose you borrow $100. The bank will have you pay back more than $100. This is how the bank makes its money, so that it can pay its help and stay in business. The extra money you pay back is called interest. If you borrow $100 and pay back $102, then you pay $2 interest.

If you have money to invest, you may earn interest on that money. If you put your money in a savings account at a bank, they will pay you interest. If you deposit $50 and the bank pays you $3 at the end of the year, you receive $3 interest.

LET'S EXPLORE

1. You borrow $100. One year later you pay back $103.
   (a) How much interest do you pay?
   (b) What is the per cent of interest?
   (c) What is 3% of $100?
2. You borrow $200. One year later you pay back $210.
   (a) How much interest do you pay?
   (b) What is the per cent of interest?
   (c) What is 5% of $200?
3. You borrow $500. One year later you pay back $520.
   (a) How much interest do you pay?
   (b) What is the per cent of interest?
   (c) What is 4% of $500?
4. You borrow $600. One year later you pay back $660.

(a) How much interest do you pay?

(b) What is the per cent of interest?

(c) What is 10% of $600?

II. Wilcox, Ex. 15, page 182.

III. Nichols, Ex. 1-10 page 180.

REFERENCES

Nichols, Eugene D., Pre-Algebra Mathematics,
Holt, Rinehart and Winston, Inc., 1970.

Wilcox, Marie S., Mathematics A Modern Approach,
Addison-Wesley Publishing Co., 1971.

Wollensak Teaching tapes:

C-3151 Introduction to Per Cent

C-3152 Changing Fractions to Per Cent

C-3155 Changing Per Cent to Fractions

C-3154 Changing Per Cent to Decimals

C-3157 Changing Decimals to Per Cent

LEARNING
ACTIVITY
PACKAGE

SETS

Ninety Six High School
Ninety Six, S.C.

Pre-ALGEBRA 93-94

REVIEWED BY

LAP NUMBER 5

WRITTEN BY Diane Evans

Instructions

I. Read Rationale

II. Read BEHAVIORAL OBJECTIVES.

III. Resources

A. All work must be done in math notebook with pencil only.

B. Keep your notebook up to date. Your teacher may ask for it at any time (without warning).

C. Work all the Exercises in at least TWO text for each goal. Always check your exercises in your notebook (see your teacher).

IV. Self-Evaluation

A. Must be taken at completion of activities for each section.

B. Does not affect your grade in any way.

V. Advanced Study

A. To be done only after all previous work has been satisfactorily completed.

B. Must be approved by teacher.

VI. Progress Test and LAP Test

A. Teacher graded

B. Recycling may take place at this time if test is not satisfactory.

DO NOT LOSE YOUR LAP. If you do, you must buy another one.

Rationale (The LAP's Purpose)

A natural place to begin the study of any branch of mathematics is in the theory of sets. Sets can be thought of as the building blocks of mathematics. Before a student can proceed to any form of higher mathematics, he must be familiar with what a set is, operations with sets, and how sets can be used.

In this LAP you will be given a systematic study of the subject of sets, including the basic notation associated with sets. Other concepts you will study are subsets, equality and matching sets, operations on sets, infinite sets, and graphing sets.

Section 1

BEHAVIORAL OBJECTIVES : At the completion of your prescribed course of study, you will be able to:

1. Given any set stated in words, write and/or identify it in set notation.

2. Given any set written in the description method, rewrite or identify it in the listing or roster method.

3. Given any set written in the listing method, rewrite or identify it in the description method.

4. Given a particular set and a list of elements, decide which are elements of that set and which are not.

5. Given any sets, tell which are infinite and which are finite.

6. Given a rule for a set (descriptive form), denote whether or not the result is the empty set - $\emptyset$, $\{\}$.

7. Given a list of numbers, be able to name those that are prime & those that are composite.

8. Given any two sets, denote whether or not they are matching sets (one to one correspondence).

Resources

I. Reading and Problems

1. Vanatta, Algebra One. #1, pp. 8-12, Ex. 1 p. 12: #2 & 3, pp. 8-12, Ex. ___: #4, 5, 6 pp. 8-12, Ex. ___: #7 ____: #8, pp. 8-12, Ex. ____.

2. Payne, Algebra One. #1, 2, 3 ___: #4,5,6 pp. 1-4 and 2 definitions on p. 13; ex. 1-10 and 38-40, pp. 4-6: #7 p. 5 read problems 16, 17, Ex. 15, 16 p. 5 and 3 p. 45: #8 pp. 12-14, Ex. 14, 15 pp. 15, 16.

3. Nichols, Modern Elementary Algebra. #1 pp. 1-3 Ex. 6 p. 3 and 8 p. 29: #2, 3 ___: #4,5,6 pp. 1-3, Ex. 1-4, 6, 9-11 p. 3 & 4: #7 pp. 4-6, Ex. 2-6 p. 6: #8 pp. 7, 8, Ex. 1, 5, p. 8-9.

4. Dolciani, Modern Algebra Bk. 1. #1 ____: #2, 3 pp. 10-14, Ex. 1-6 and 7-26 even on p. 12, and 1, 2, 8, 9, 12, 13, 16 written (roster only) p. 14: #4,5,6 pp. 13-14, Ex. 2, 4, 6, 8, 10, 12 (omit roster) p. 14, 15, and 17, 18, 19, p. 27: #7 ___: #8 pp. 13-14, Ex. 9-16 oral.

5. Wooton, Modern School Mathematics. #1,2,3 ___: #4,5,6 pp. 16-18, Ex. 31-36, p. 19: #7 ___: #8 pp. 20-23, Ex. 27, 28, p. 25.

6. Pearson, Modern Algebra. #1, 2, 3 ___: #4,5,6 pp. 1-4, Ex. 1-10 and 38-40 p. 4, 5, 6: #7, 8 ___.

7. Introduction to Sets (Programmed) #4, 5, 6 frames 1-67: #8 frames 226-255.

II. Audio

Wollensak C-3451 Introduction to Sets

Wollensak C-3010 Prime Numbers

Math DE-1 Sets: Matching, equal, disjoint

III. Games

On-Sets by Layman Allen

Self-Evaluation

OBJECTIVE

1 I. Write the following sets in set notation:

_____ 1. The set of vowels in the word algebra.

_____ 2. The months of the year whose names begin with D.

_____ 3. The odd numbers between 5 and 11.

_____ 4. The states that have a boundary in common with S.C.

_____ 5. The even numbers between 8 and 12.

_____ 6. All numbers divisible by 25 less than 100.

2 II. Rewrite these sets using the listing method.

_____ 7. {All whole numbers less than 4}

_____ 8. {capitals of S. C., Ga. N.C.}

_____ 9. {colors in the American flag}

_____ 10. {even nos. less than 10}

_____ 11. {nos. less than 20 that are divisible by 6}

3 III. Rewrite these sets using the description method.

_____ 12. {2, 4, 6}

_____ 13. {Washington, D. C.}

_____ 14. {Lyndon Johnson, Richard Nixon}

_____ 15. {5, 10, 15, 20, 25}

_____ 16. {1, 3, 5, 7, 9}

4 IV. Match the set on the left with an element on the right not in the set.

____ 17. $\{1, \frac{1}{2}, 3, 4, 9\}$

a. 1
b. $\frac{2}{4}$
c. 0
d. $\frac{24}{48}$
e. $\frac{16}{4}$

____ 18. $\{1, 2, 3. . .\}$

a. $\frac{4}{2}$
b. 5
c. 2
d. $\frac{1}{2}$
e. 1

____ 19. {all the numbers larger than five and less than 2.}

a. 3
b. 0
c. 6
d. $\frac{1}{3}$
e. none

5, 6 V. Classify the following sets as being either infinite, finite, or empty (Ø), { }.

____ 20. the set of odd natural numbers

____ 21. the set of even natural numbers

____ 22. the set of prime numbers

____ 23. the set of prime numbers greater than 10 and divisible by 2.

____ 24. the set of even prime numbers

____ 25. the set of odd prime numbers

____ 26. the set of even numbers that are odd

7 VI. Classify the following numbers as being either prime or composite.

____ 27. 13

____ 28. 51

____ 29. 9

____ 30. 6

____ 31. 101

8 VII. Given the following pairs denote whether or not they are matching (one to one correspondence).

____ 32. $A = \{2, 5\}$

$B = \{1, 3, 8\}$

SELF-EVALUATION (cont')

____ 33. $A = \{0\}$

$B = \{1, 2, 3\}$

____ 34. $A = \{1, 2, 3, 4...\}$

$B = \{1, 3, 5, 7...\}$

____ 35. $A = \{50, 49, 48 \ldots 3, 2, 1\}$

$B = \{50, 51, 52 \ldots 97, 98, 99\}$

IF YOU HAVE MASTERED ALL YOUR BEHAVIORAL OBJECTIVES, TAKE THE PROGRESS TEST. CONSULT YOUR TEACHER FIRST.

Section 2

BEHAVIORAL OBJECTIVES: At the completion of your prescribed course of study, you will be able to:

9. Determine whether or not a set is a subset of a given set.
10. Given a set and all its subsets, name those subsets which are proper subsets of the given set.
11. Given a pair of sets, determine whether or not they are equal.
12. Given a pair of sets, determine if it is disjoint.
13. Given any pair of sets, name their union and intersection in the indicated manner.
14. Given a universal set with any number of elements, determine its number of subsets.
15. Given any set which is a subset of a universal set, name the set which is its complement.
16. Given a Venn diagram, choose from a list the operation(s) being illustrated.

Resources

I. Reading and Problems

1. Vanatta, Algebra One. #9, pp. 13-14, Ex. 2, 4 p.15: #10 p. 14, Ex.6, 7 p. 15: #11, p. 9, Ex. 8 p. 12: #12, p. 16 Ex. 2 p. 21: #13 pp.15-18 Ex. 5a,b, p.19 and 2a,b, p.20: #14 ____: #15,16,pp.14-18 Ex. 1,2,3 p. 18.

2. Payne, Algebra One. #9, 10, 11, 12 pp. 6-9 and 12-14 Ex.1-7, 18-47 pp. 9, 10 and 1-10 p. 15: #13, 14 ____: #15, 16 p. 7 read example 4, Exer. 5-14 p. 45.

3. Nichols, Modern Elementary Algebra. #9, 10, 11, 12 pp. 9-14 Ex 1,2,4,5,6 p. 11 and 1, 2, 3, 4 pp14, 15: #13 pp. 15-17, Ex.1, 2, 9, 10, 11, 12 pp. 17-18: #14 pp. 18-21 Ex 1-6 p. 20: #15, 16 pp. 22-25 Ex 1-7 pp. 22-25.

4. Dolciani, Modern Algebra Bk 1. # 9,10,11,12 p. 18 Ex. 1-10 p. 18 and 9-14 p. 19: #13, 14, 15, 16 ____.

5. Wooton, Modern School Mathematics. #9, 10, 11, 12 pp. 20-23 Ex. 11-18 p. 24 and 17-20 p. 25: #13,14,15, 16 ____.

6. Pearson, Modern Algebra. #9, 10, 11, 12 pp. 2, 5-7 Ex. 5p3 and 1-4, 6 p7: #13 pp. 10-12 Ex. 1a, b, 3, 4 a, b pp. 12-13 and 12 p 24, and 6 p 25: #14 ____: #15, 16 pp. 9, 12 Ex.1, 2, pp. 9-10 and 6, 7, 10 p. 13.

7. Introduction to Sets (Programmed) #9, 10, 11, 12 frames 86-144: #13 frames 164-194: #14 frames 145-161: #15,16 frames 195-225.

II. Audio

Wollensak C-3451 Introduction to Sets

Wollensak C-3452 Venn Diagrams and Number Line

Wollensak C-3602 Intersection and Union: Points

Math DE-2 Subsets, proper subsets

III. Games

On-Sets by Layman Allen

Crossword Puzzle on Sets

A Set Code

A Set Cartoon

Self-Evaluation 2

OBJECTIVES

11,12 I. Indicate whether the following statements are true or false.

_____ 1. The set of all even natural numbers and the set of all prime numbers are disjoint sets.

_____ 2. If $A \subset B$ and $B \subset A$, the $A = B$.

_____ 3. The set of all odd natural numbers and the set of all even natural numbers are disjoint sets.

_____ 4. If $A = \{1, 2, 3\}$ and $B = \{1 + 2, 1 + 1, 1 + 0\}$, then $A \neq B$.

_____ 5. $\{1, 2, 3\} = \{\{1\}, \{2\}, \{3\}\}$

13 II. Give the union and intersection of the following pairs of sets.

_____ 6. $A = \{2, 5\}$ $B = \{1, 3, 8\}$

_____ 7. $A = \{0\}$ $B = \{1, 2, 3\}$

_____ 8. $A = \{1, 2, 3, 4...\}$, $B = \{1, 3, 5, 7...\}$

_____ 9. $A = \{2, 4, 6, 8...\}$ $B = \{1, 3, 5, 7...\}$

_____ 10. $A = \{50, 49, ..2, 1\}$ $B = \{50, 51, ...99, 100\}$

_____ 11. $A = \{2, 5, \{6\}, 8\}$ $B = \{2, 5, 6, 10\}$

3,14,15 III. Indicate whether the following sentences are true or false.

_____ 12. If N = the set of all natural numbers and P = the set of all prime numbers, then $N \cup P = P$.

_____ 13. If E = the set of all even natural numbers, G = the set of all odd natural numbers, and N = the set of all natural numbers, then $E \cap G = N$.

_____ 14. If $A = \{0, 1, 2\}$, then A has eight subsets.

_____ 15. If $B = \{1\}$, then B has one subset.

_____ 16. $\forall$ sets A $\quad A \cap \overline{A} = A$

_____ 17. $\forall$ sets A $\quad A \cup \overline{A} = \emptyset$

_____ 18. {3,6,9,12} has 8 subsets.

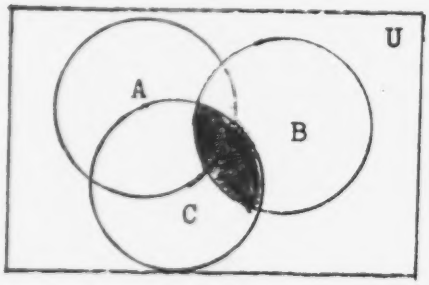
_____ 19. The intersection of the set of prime numbers and the set of even numbers is the empty set.

_____ 20. Given a set of 3 elements, there are exactly 7 <u>proper</u> subsets of that set.

_____ 21. Given any infinite set, there exists only a finite number of subsets.

16 IV. In the following Venn diagrams, indicate the name for the sets denoted by the shaded AREA.

___ 22.



The Shaded Area is

a. $A \cup B$

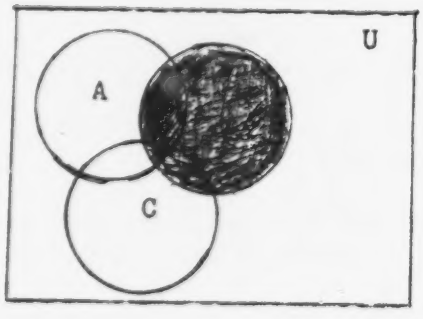
b. $B \cup C$

c. $A \cup C$

d. $B \cap C$

e. $A \cap C$

___ 23.



The Shaded Area is

a. $B \cap A$

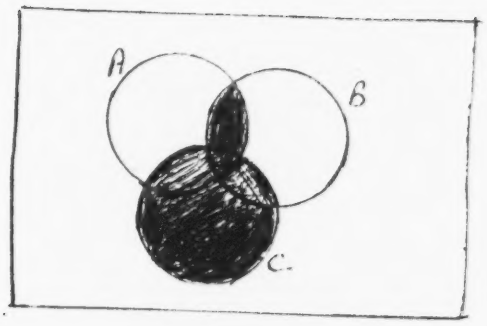
b. $A \cap B$

c. $B \cup A$

d. $(A \cup B) \cap C$

e. none of these

___ 24.



The Shaded Area is

a. $(A \cup B) \cup C$

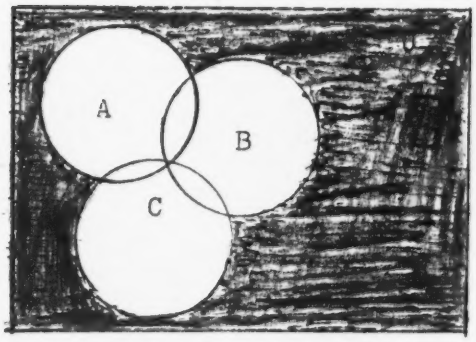
b. $(A \cap B) \cup C$

c. $A \cup C$

d. $(A \cup B) \cap C$

e. none of these

___ 25.



The Shaded Area is

a. $\overline{(A \cup B) \cup C}$

b. $A \cap \overline{(B \cup C)}$

c. $(\overline{A} \cap \overline{B}) \cup \overline{C}$

9, 10 VIII. Indicate whether the following statements are true or false.

_____ 26. If $A \subseteq B$, then every member of B is also a member of A.

_____ 27. $\forall$ sets A, $A \subseteq A$

_____ 28. $\forall$ sets A, $\emptyset \subseteq A$

_____ 29. If $A = \{0, 1\}$, then there is only one subset of A which has exactly two elements.

_____ 30. Exactly one subset of the set of natural numbers has no elements.

_____ 31. $3 \subset \{3,4,5,6\}$

IF YOU HAVE MASTERED ALL YOUR BEHAVIORAL OBJECTIVES, TAKE THE LAP TEST. CONSULT YOUR TEACHER FIRST.

Advanced Study

The following is an application of set theory.

1- Take either kingdom and draw a venn diagram showing proper subsets. Do the same with one particular species. The most interesting may be Sapiens.

2- Color TV is based on the idea that there are three primary colors, each transmitted separately and then blended in your set circuits at home. Use a Venn diagram of the three primary colors to show how the colors are blended. Use set notation to describe the different color obtained.

3- Given the following quadrilaterals construct a Venn Diagram Showing their proper realtionships.

Quadrilaterals

Rhombus

Square

Parallegagram

Rectangle

Trapazoid

4- Find the prime factorization of 10,001.

References

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Dolciani, Mary P., Wooten, William, Beckenbach, Edwin F., Jurgensen, Ray C., Donnelly, Alfred J., Modern School Mathematics, Algebra 1,

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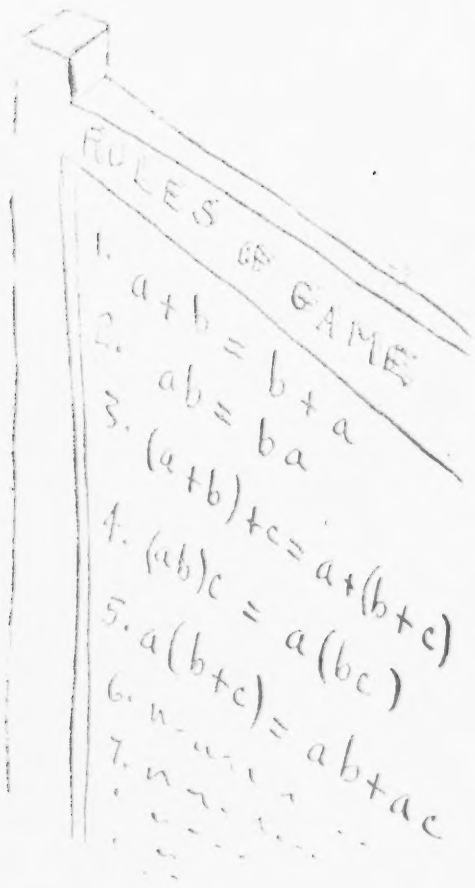
Audio Visual

Wollensak Teaching Tapes: C-3451 C-3602 C-3010 C-3452

L EARNING

A CTIVITY

P ACKAGE



PROPERTIES OF OPERATIONS

Ninety Six High School - Ninety Six, S.C.

Pre-ALGEBRA 93 - 94

REVIEWED BY

LAP NUMBER 26

WRITTEN BY Diane Evans

RATIONALE (The LAP's Purpose)

Mathematics can be thought of as a game in which you perform moves by specific rules. For example:

WHAT NUMBER IS EQUAL TO $4 + 3 \times 5$?

You may work it out this way:

$4 + 3 = 7 \times 5 = 35$

or you may compute it this way:

$3 \times 5 = 15$ and $4 + 15 = 19$

As you can see there are two possible answers!! Obviously, both ways cannot be correct because 35 is not equal to 19! The expression $4 + 3 \times 5$ must have only one meaning! It is customary to use parentheses, which are mathematical punctuation marks, to make the meaning of such phrases clear.

In this LAP you will not only learn how parentheses are used but also about the "order of operations" - that is the order in which certain operations are to be performed. Though you have studied the properties of operations in the past, you will review them again because of their importance in learning mathematics.

Behavioral Objectives

Upon completion of your prescribed course of study, you will be able to:

1. Write the simplest name for any numerical expression which invol the use of grouping symbols and order of operation.

2. Given any mathematical sentence, identify which of the following properties (if any) are being illustrated.

   a) The Commutative Property of Addition (CPA)

   b) The Commutative Property of Multiplication (CPM)

   c) The Associative Property of Addition (APA)

   d) The Associative Property of Multiplication (APM)

   e) "Symmetric Property of Equality" (SPE)

   f) "Distributive Property of Multiplication over Addition" (DPMA

3. Given any set, determine if it is closed with respect to a given operation

4. Given any mathematical sentence, identify which of these propertie (if any) are being illustrated.

   a) Multiplicative identity

   b) Property of one for Division (P1D)

   c) Additive identity

   d) Property of Zero for Multiplication (PZM)

5. Given any mathematical sentence, identify which of the following properti is being illustrated:

   a. multiplicative inverse
   b. additive inverse

6. Given any mathematical sentence involving one operation, write an equivalent sentence using the inverse operation.

7. Given any word phrase, of the type in Appendix I, translate it into an equivalent mathematical phrase.

8. Correctly write a mathematical sentence of the type in Appendix II which would be used to solve a given word problem.

RESOURCES

I. Reading and Problems. (EOL means every other letter)

1. Vanatta, Algebra One, #1 pp.46-47, Ex. 1-4 p. 47: #2 pp. 27-33, Ex. 2,7,10 p. 29, 13, 18, 19, 24 p. 34, 14,15 p. 37, 14 p. 30, 6,20 p. 34: #3 pp. 27 study 1, p. 28 study 1, Ex. ___: #4,5 pp. 27 study 2-5, p. 28 study 2-5, Ex. 1,4,8,9 p. 29, 12,13, p. 30 15-17, 21,22, p. 34, 17,17,19 p. 37: #6 ___: #7 ___: #8 pp154-156, 1-10,12-15 pp. 156-157.

2. Dolciani, Modern Algebra, Book 1, #1 p. 23, Ex. 18,19,20,30-34, p.24: #2 pp. 69, 73-76; Ex. 1-26 p. 74; 15,16 pp. 100-101; 1,6, 7, p. 70; 2 p. 98: #3 pp. 70-71, Ex. 1-14 oral p. 72, 1-12 written p. 73: #4,5 pp. 77, 121,138, Ex. 33-42, p. 141, 1-10 oral p. 122: #6 ___: #7 p. 51, Ex. 1-22 pp. 52-53, 1-25 even p. 54, 3,6,13,14 p. 55: #8 pp. 56-57, Ex. (write equation only) 1-14 odd pp. 57-58, 1-14 p. 89.

3. Nichols, Modern Elementary Algebra, #1 pp31-33, Ex. 1-25 odd pp. 33-34, 2 EOL p. 37: #2 pp. 34-37, 40-46, Ex. 1 p. 37, 1 EOL, 2 pp. 40-41; 1, 2 EOL, 3 p. 42-43: #3 pp. 37-39, Ex. odds p. 39, 4a,c,e p. 51, 8(state why or why not) p. 53: #4 pp. 43-45, Ex. 2 p. 45: #5 p. 105 Ex. ___: #6 pp. 46-48, #x. 1-3 EOL pp. 47-48: #7 pp. 48-49, Ex. 1-21 p. 49: #8 ___.

4. Payne, Algebra One, #1 p. 18, Ex. 1-5 p. 18: #2 pp. 31-38, Ex. 1-3 odd p. 33, 1-51 odd pp. 35-37: #3, p. 31, Ex. 62-73 pp. 37-38: #4 pp. 31-38, Ex. ___: #5 pp. 63-65, 75 Ex. 1-20 p. 65: #6 ___: #7 ___: #8 pp. 65-66, Ex. 21-23 pp. 65-66.

5. Dolciani, Modern School Mathematics, #1 pp. 10-16, Ex. 1-10, 25-32 pp. 13-14, 1-10 odd, 17-29 odd pp. 15-16, 1-10 p. 51: #2 pp. 48-52, 55-59, 71-76, Ex. 11-28 pp. 50-51, 1-6, 7-23 odd pp. 57-58: #3 p. 47, Ex. 1-10 pp. 49-50: #4 p. 61, 82-87, Ex. ___: #5 pp. 66-70, 76-81, Ex. 1-10: #6 ___: #7 Ex, 11-24 p. 13, 11-16, p. 16: #8 pp. 29-33, Ex. 31-38 p. 33 (equation only).

6. Pearson, Modern Algebra A Logical Approach, #1 pp. 52-59, Ex. 1-25 odd p. 55, 1-3 EOL pp. 56-57, 1-5 EOL p. 58: #2 pp. 139-145, 166-169, Ex. 1-14 pp. 141-142, 1-3,4 EOL, 5,6 pp. 144-145,1-4 EOL, 5-7 pp. 168-169: #3 pp. 135-137, Ex. 1-12 pp. 136-137: #4, pp. 162-166, 178-180, Ex. 1,2 EOL, 3,5-7 EOL, 9 pp. 164-166: #5 p. 204, 162-163, Ex. 2 a,b,e,f,g,5a-h p. 165, 3a-e, 4a-d, g-i p. 296: #6 ___: #7 pp. 145-147, Ex. 10-13, p. 147: #8 pp. 157-159, Ex. (write equations only), 1-19 odd pp. 158-159, 5-15 odd pp. 176-177.

7. #7 Appendix 1

8. #8 Appendix 2

II. Video-Tapes

Wollensak C-3453 The Commutative Property
C-3454 The Associative Property
C-3455 The Distributive Property

Wollensak C-3456 The Closure Property
C-3459 Identity Element
C-3451 The Inverse Element
C-3801 Open Phrase, Open Sentence
C-3803 Open Sentence: Sclution
C-3809 Reading Written Problems

III. Games

Equations by Layman Allen

APPENDIX I

1. What is the cost of n pencils at 3 cents each?

2. What is the cost of x articles at y dollars each?

3. How far can a boy run in h hours at the rate of 6 miles per hour?

4. The sum of two numbers is 7 and one of them is x; what is the other number?

5. Represent in terms of x two numbers that have the ratio 3:4.

6. If x represents the sum of two numbers and one of them is 5, what is the other?

7. What is the total weight of n boys weighing y pounds each?

8. The sum of two numbers is x and one of the numbers is 5. What is the other number?

9. How many cents are there in d dollars?

10. Mary is n years old now. How old was she 3 years ago?

11. How many inches are there in x feet and five inches?

12. A man had x dollars and spent y dollars. How much did he have left?

13. If n represents a certain number, represent the next larger consecutive number.

14. A parcel weighs t pounds and a smaller parcel weighs $\frac{2}{3}$ as much. What is the weight of the smaller parcel?

15. What is the average weight of two boys who weigh x pounds and y pounds each respectively?

16. How much salt remains when x pounds have been used from a bag containing a pounds?

17. What is the perimeter of a square one side of which is s?

18. The difference between two numbers is 2 and the smaller number is n. What is the other number?

19. One part of t is w. What is the other part?

20. Elizabeth's age is now 7 years. How old will she be in n

21. $x = 5y$. Upon what does the value of x depend?

22. What is the perimeter of a triangle whose sides are a, b, a

23. What is the perimeter of a rectangle whoe length is l and whose width is w?

24. What is the area of a rectangle whose base is b and whose height is h?

25. How many inches are there in y yards, f feet and i inches?

26. If $2n + 1$ represents an odd integer, write an algebraic expression for the next larger odd integer.

27. If $y = \frac{8}{x}$, does y increase or decrease as x increases?

28. A customer purchases n pounds of sugar at c cents a pound and gives in payment a one-dollar bill; express in cents the change he should receive.

29. In a certain freshman class there are a boys and b girls. press the ratio of the number of girls to the total number of pupils.

30. A dealer sold n automobile tires at s dollars each and there gained r dollars. What was the cost of the tires?

31. A man buys 3 lemons at c cents each and t oranges at d cents each. How much does he pay for them all?

32. If a man walks on top of a freight train toward the engine at the rate of 3 miles per hour, and the train moves at the rate of k miles per hour, at what rate does the man pass a telegraph pole?

33. If a man can row in still water at the rate of y miles an hour how fast can he row up a stream whose rate of flow is x miles an hour?

34. If a train travels 314 miles at the rate of y miles per hour, how long does it travel?

35. If k apples can be bought for c cents, how much will m apples cost?

36. If a man earns d dollars a month and saves c dollars a month, how much does he spend in a year?

37. Mary is x years old and her father is three times as old. How old was her father 8 years ago?

38. A ship sals r miles the first day, s miles the second day and t miles the third day. Express the average daily rate.

39. If A travels x miles an hour and B y miles an hour, write an equation that expresses the fact that in 5 hours A travels the same distance that B travels in 6 hours.

40. A man bought a suit for d dollars and received four dollars in exchange. How much money did he pay the clothier?

C. Applications

1. If the pressure of a gas at constant temperature is doubled, the volume ("V") of a gas changes inversely proportional to the change in pressure ("P").

2. The energy available to any trophic level is equal to the biomass of the next lower trophic level minus the loss of energy due to transfer inefficiency. If the trophic level is "x" and the loss of energy due to transfer inefficiency is "y", what is the biomass of the next lower trophic level "b"?

3. When a 2N chromosome number is halved during meiosis, the resulting gametes have the N number of chromosomes. How many chromosomes would there be in a zygote which is formed by the fusion of 2 gametes?

4. If 50 field mice occupy 10 acres of pasture, what is their density per acre in that pasture? (mice per acre)

5. The actual size of an object is q microns. If the microscope magnifies 100 times, what will the apparent size be?

6. The ocular lens magnifies r times and the objective lens magnifies s times. What is the magnifying power of this compound microscope?

7. The harvestable crop of a wild-life population is equal to the excess number of animals above the carrying capacity of the habitat. If the population size of white-tailed deer is x and the population size carrying capacity of the habitat is 1000, what is the number of animals above the carrying capacity?

Write the mathematical sentence which would be used to solve each word problem.

1. Five times a certain number is 105.

2. The greater of two numbers is twice the smaller and their sum is 48.

3. Mary is 5 times as old as her brother and their combined ages total 18 years.

4. One number is 4 times another and their difference is 24.

5. A man walks 6 hours at a certain rate and then proceeds 3 hours at twice his former rate. If he walked 24 miles in all, at what rate did he start walking?

6. The length of a rectangle is three times its width and its perimeter is 56 feet.

7. The sum of three numbers is 56. The second number is 3 times the first and the third number is 4 times the first. What are the numbers?

8. The sum of the three angles of any triangle is 180 degrees. In a certain triangle ABC, angle A is twice as large as angle C and angle B is three times as large as angle A.

9. Two numbers have the ratio 5:6 and their sum is 88. Find the numbers.

10. Separate 92 into two parts such that one part is three times the other.

11. One number is three times another. Six times the larger diminished by twice the smaller is 48.

12. When a quart of cream cost four times as much as a quart of milk, 5 quarts of milk and 3 quarts of cream cost $2.72, what is the cost of each per quart?

C. Applications

1. If you mix 4 grams of salt and 84 grams of water, what percent of the total solution is salt?

2. Compound X is composed of elements Y and Z in the ratio of 3:2. If you had 50 grams of element Y, how many grams of element Z would you need to utilize all of Y into making compound X?

Self-Evaluation Test

Behavioral Objectives

1 I. Write the simplest answer for each of the following:

____1. $56 \div 2 \times 3 \div \frac{1}{2}$

____2. $(\frac{3}{4} - \frac{1}{3})(3 + 9) + 3 \quad [(5 - 3) \div 6]$

____3. $15 - 3 \times 2 + (18 - 5) \cdot 5$

____4. $\frac{3}{5} \div (2 + \frac{1}{3}) + (\frac{3}{7} \times \frac{2}{5})$

____5. $5 \cdot 0 + 4 - 2 \cdot 2$

____6. $[5 \cdot (2) + 11] \div 7$

____7. $4 \times \{78 - [5 \cdot 4 + (15 \div 3)] \times 3\} \cdot 2$

____8. $28 \div 2 - 2 \times 6 \div 3 + 3$

____9. $(7 + 5)^2 - 6$

____10. $7 \times 8 - 6$

2 II. For each given sentence, write the name of the property illustrated. (Use abbreviations such as CPA, etc.)

____11. $(46 + 21) + 7 = 46 + (21 + 7)$

____12. $16 \cdot (5 + 7) = (5 + 7) \cdot 16$

____13. $4 \cdot (7 + 13) = 4 \cdot (13 + 7)$

____14. $15 \cdot (6 \cdot 2) = (15 \cdot 6) \cdot 2$

____15. If $a + b = c$ then $c = a + b$

____16. $4 \cdot [x + (2 + 5)] = 4 \cdot x + 4 \cdot (2 + 5)$

____17. $ax + ay = a \cdot (x + y)$

3 III. Each of the following statements is either true or false.

____ 18. The set $\{0, \frac{1}{2}, 1\}$ is closed under multiplication.

____ 19. The set of odd natural numbers is closed under the operation of multiplying by the factor 2.

____ 20. The set of even natural numbers is closed under addition.

____ 21. The set $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots\}$ is closed under addition.

____ 22. The set of all prime numbers is closed under addition.

4 IV. For each given sentence, write the name of the property illustrated. (Use abbreviations such as P1 M, etc.)

____ 23. If $3 + 7 = n$ then n is a natural number.

____ 24. $y \cdot (5 - 4) = y$

____ 25. $5 \cdot (x - x) = 0$

____ 26. $x + 0 = x$

____ 27. $7 + 0 = 7$

____ 28. $6 \cdot 1 = 6$

6 V. For each statement, write a correct related problem using the inverse operation.

_____ 29. $11 - 4 = 7$

_____ 30. $\frac{1}{10} + \frac{1}{2} = \frac{3}{5}$

_____ 31. $51 \div 3 = 17$

_____ 32. $\frac{15}{23} \times \frac{7}{5} = \frac{3}{4}$

7 VI. For each phrase write a correct mathematical phrase.

_____ 33. The sum of four times n and 7.

_____ 34. 15m decreased by one-half x.

_____ 35. Four y divided by the sum 5 and x.

_____ 36. The quotient of seven x and fifteen.

_____ 37. Three times the difference of 2 and q.

_____ 38. The product of 5 and x increased by t.

_____ 39. Seven less than four p.

_____ 40. Four times the sum of n and 7.

8 VII. Translate each word sentence into an equivalent open mathematical sentence. (Do not solve)

_____ 41. The difference of three times a number (w) and six is the number increased by four.

_____ 42. The quotient of seven and the sum of a number (x) and eight plus five is twenty-three.

_____ 43. One-fifth of a certain man's lifetime (y) spent in childhood, plus one-third of his life served in the armed services, totals the eight years he wasted as a bum and the two-fifths of his life as a missionary.

5 VIII. For each given sentence, write the name of the property illustrated.

_____ 44. $2 \cdot \frac{1}{2} = 1$

_____ 45. $6 + (-6) = 0$

_____ 46. $\frac{2}{3} \cdot \frac{3}{2} = 1$

_____ 47. $(-7) + 7 = 0$

_____ 48. $8 \cdot \frac{1}{8} = 1$

ADVANCED STUDY

1. Dolciani, Modern Algebra, Page 90 nos. 21-31. (Work at least six problems)

2. Dolciani, Modern Algebra, page 95 nos. 17-21. (Work at least 4 problems)

3. Dolciani, Modern School Mathematics, page 52 nos. 17-20. (Work all problems) .

4. Research the concept of field - Select a system of numbers and determine if the system is a field. Write a report on your findings, giving reasons for your conclusions.

5. Prepare a bulletin board showing all the properties and their relationships to the following sets of numbers: naturals, wholes, integers, and rationals.

References

1. Vanatta, Glen D. and Goodwin, A. Wilson, Algebra One: A Modern Course, Charles E. Merrill Publishing Co., 1966.

2. Dolciani, Mary D., Berman, Simon L., and Freilich, Julius, Modern Algebra, Book 1, Houghton Mifflin Co., 1965.

3. Nichols, Eugene D., Modern Elementary Algebra, Holt, Rineha and Winston, Inc., 1965

4. Pearson, Helen R., and Allen, Frank B., Modern Algebra A Logical Approach, Ginn and Company, 1964.

5. Payne, Joseph N., Zamboni, Floyd F., Lankford, Jr., Franc Algebra One, Harcourt, Brace and World, Inc., 1969.

6. Dolciani, Mary P., Wooton, William, Beckenback, Edwin F., Jurgensen, Ray C., Donnelly, Alfred J., Modern School Mathe Algebra 1, Houghton, Mifflin Company, 1967.

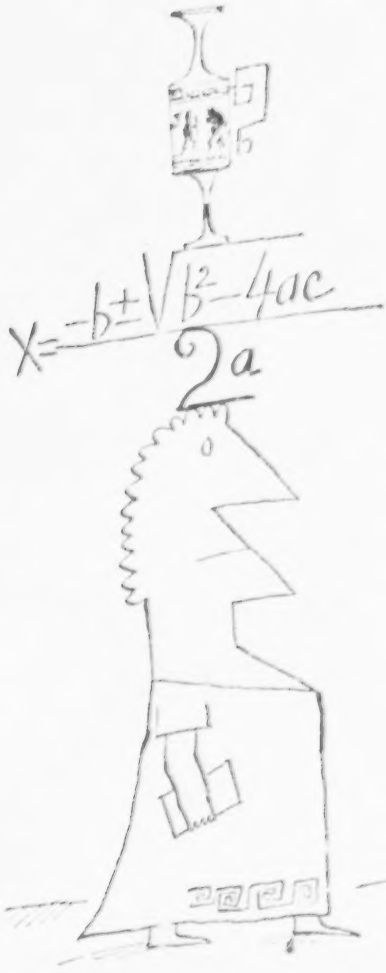
7. Wollensak Teaching Tapes

   Wollensak C-3453 The Commutative Property
   C-3454 The Associative Property
   C-3455 The Distributive Property
   C-3456 The Closure Property
   C-3459 Identity Element
   C-3451 The Inverse Element
   C-3801 Open Phrase, Open Sentence
   C-3803 Open Sentence: Solution
   C-3809 Reading Written Problems

8. Equations by Layman Allen

RATIONAL NUMBERS

LEARNING
ACTIVITY
PACKAGE



Ninety Six High School
Ninety Six, S.C.

PRE ALGEBRA 93-94

REVIEWED BY _____

LAP NUMBER 8

WRITTEN BY _____

Acknowledgement

The administration and staff of Ninety Six High School gratefully acknowledges the assistance provided by the staff of Nova High School, Fort Lauderdale, Florida. We are especially indebted to Mr. Lawrence G. Insel and Mr. Laurence R. Nantuck of Nova's Math Department for permitting us to use much material developed by them, some of which has been reproduced in its original form.

RATIONALE:

You have studied many sets of numbers through your mathematical career. The first set you discussed was the set of "NATURAL NUMBERS" (1, 2, 3, . . .). You then added zero and the set became the "WHOLE NUMBERS", after which you extended the set to include additive inverses and the set became the "INTEGERS". When you finally added the multiplicative inverses and arrived at the "RATIONAL" numbers, it appeared as though you were finished!

In this LAP you will extend the set of "RATIONAL" numbers. We will call the numbers that we ADD, the "IRRATIONAL" numbers. The set then becomes the "REAL" number system. Once you have at your disposal knowledge of the complete set of REAL numbers you will be equipped to investigate the basic concepts of elementary Algebra!

SECTION I

Behavioral Objective

After having completed your prescribed course of study, you will be able to:

1. Write or identify the definition of the sets of natural numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers.

2. Given any number, determine if it is a member of the set of

   a. natural numbers
   b. whole numbers
   c. integers
   d. rational numbers
   e. irrational numbers
   f. real numbers

3. Given any statement involving relationships among the sets of natural, whole, integer, rational, irrational, and real numbers, determine if it is true or false.

4. Determine if each of the sets of natural, whole, integer, and rational numbers is a field. If a set is not a field, state the properties that do not apply. Appendix I will be completed and turned in to the teacher.

5. Given a pair of integers a and b, determine whether $a < b$, $a = b$, or $a > b$.

6. Given two or more rational numbers, compute their sum, difference, quotient, and/or product.

RESOURCES

I. Reading and Problems. NOTE: EOL means every other letter.

1. Vanatta, Algebra One, #1, pp. 22, 80-82, Ex. 1-5 p. 83: #2, pp. 79-83, Ex. 14 p. 84: #3, pp. 79-83, Ex. 13 p. 83: #4, pp. 28-29, 85, Ex. 2,3,5 p. 86: #5 ___: #6 pp. 90-106, Ex. 1-13 pp. 92-93, 1-24, p. 95, 1-16 p. 99, 1-20 odd (bottom) p. 100-101, 1-26 p. 104, 1-18 p. 106, 21-25 p. 107.

2. Dolciani, Modern Algebra, Book 1, #1,2,3,4,5 ___: #6 pp. 125-126, 128-130, 133-135, 138-140, Ex. 1-16 p. 126, 1-12 p. 130, 1-11 p. 135, 1-12 p. 140, 1-8 p. 141.

3. Nichols, Modern Elementary Algebra, #1,2,3 ___: #4 pp. 104-105, Ex. 1-3 p. 105: #5 pp. 55-59, Ex. 1-5 EOL pp. 59-61, 1-2 EOL p. 62: #6 pp. 62-63, 85-88, 92-94, 96-98, Ex. 1-2 EOL p. 64-65, 6 a - n p. 68, 3a-z p. 89, 1 a-y p. 95, 2 all p. 99.

4. Dolciani, Modern School Mathematics, Algebra I, #a pp. 22, 316, 425, Ex. ___: #2,3,4 ___: #5 pp. 5-8, Ex. 1-49 odd pp. 9-10: #6 ___.

5. Payne, Algebra One, #1 pp. 2, 52-54, Ex. ___: #2 pp. 52-54, Ex. 1-12 p. 54, 1-15, 32-41 p. 55: #3 pp. 52-54, Ex. 13-20 p. 54, 16-30 p. 55: #4 pp. 535, 528, Ex. ___: #5 pp. 56-57, Ex. 1-61 odd pp. 57-58: #6 pp. 66-69, 73-74, 75-77, 79-80, 82-83, Ex. 1-51 even p. 69, 1-34 p. 74, 1-30 pp. 77-78, 10-24 p. 81, 1-12 p. 83.

6. Pearson, Modern Algebra, Book One, #1,2,3,4 ___: #5 pp. 37-38, 1-6 EOL pp. 38-39: #6 ___.

7. Arithmetic of Directed Numbers, A programmed unit, #6.

8. Supplementary LAPs-Integers, Rational Numbers, D. Evans, Ninety Six High School.

II. Audio

Wollensak C-3458 The Real Number System
C-3331 Directed Numbers: Addition
C-3332 Directed Numbers: Subtraction
C-3333 Directed Numbers: Multiplication
C-3334 Directed Numbers: Division

III. Visual - filmstrips

Comparing Fractions: Adding and Subtracting

Multiplying Fractions

Multiplication of Signed Numbers

Dividing Fractions

IV. Games

The Conversion Game by Evans

Equations by Layman Allen

Crossword Puzzle - "A Short Review of Fractions"

SELF-EVALUATION 1

1 I. Write the definition or set for each of the following:

1. natural numbers

2. whole numbers

3. integers

4. rational numbers

5. irrational numbers

6. real numbers

2 II. Identify the following numbers as elements of naturals (N), wholes (W), integers (I), rationals (Q), irrationals (Z), or Reals (R). List all the sets that contain each number.

____ 7. 28

____ 8. $\sqrt{9}$

____ 9. $\frac{3}{4}$

____ 10. $\sqrt{7}$

____ 11. $1.\overline{3}$

____ 12. .010010001...

____ 13. 0

____ 14. -18

____ 15. $-\frac{7}{8}$

3 III. True or False.

_____ 16. The natural numbers are a subset of the whole numbers.

_____ 17. The integers are a subset of the natural numbers.

_____ 18. The whole numbers are a subset of the rationals.

_____ 19. The rational numbers contain the integers and the fractions.

_____ 20. The natural numbers are a subset of the rational numbers.

_____ 21. The integers are not a subset of the rational numbers.

3 IV. True or False.

N = set of natural numbers
W = set of whole numbers
I = set of integers
Q = set of rational numbers
Z = set of irrational numbers
R = set of real numbers

_____ 22. $Q \subset R$

_____ 23. $Q \cup Z = R$

_____ 24. $W \subset N$

_____ 25. $Z \subset R$

_____ 26. $N \subset R$

4 V. Is each of the following sets a field? If no, write the properties necessary to make it a field.

_____ 27. whole numbers

_____ 28. integers

_____ 29. natural numbers

_____ 30. rational numbers

5 VI. In each blank write <, >, or = to make a true statement.

31. 7 ____ 2

32. -10 ____ 5

33. 0 ____ 18

34. 8 + 1 ____ 9

35. 0 ____ -17

36. -2 ____ 2

37. 3 · 5 ____ -15

38. -5 ____ 3

39. -7 ____ -9

40. 10 ____ -10

6. VII. Work the following:

_____ 41. $-\frac{2}{3} + -\frac{4}{5} =$

_____ 42. $-\frac{2}{3} \cdot \frac{4}{5}$

_____ 43. $-\frac{4}{5} + -\frac{2}{3}$

_____ 44. $-\frac{6}{5} + \frac{3}{4}$

80

_____ 45. $-\frac{2}{3} + \frac{4}{5}$

_____ 46. $-\frac{4}{7} + -\frac{3}{4}$

_____ 47. $-\frac{2}{5} \cdot \frac{3}{4}$

_____ 48. $-\frac{1}{6} + -\frac{2}{3}$

_____ 49. $\frac{1}{4} - \frac{1}{7}$

_____ 50. $\frac{2}{5} + -\frac{4}{5}$

_____ 51. -8×-7

_____ 52. $9 + -3$

_____ 53. $-6 - 9$

_____ 54. $28 + -7$

_____ 55. $-18 + -9$

_____ 56. $36 \cdot -2$

_____ 57. $-6 + -12$

_____ 58. $3 - -7$

_____ 59. $-18 - -2$

_____ 60. $12 + -13$

SECTION 2

Behavioral Objective

After having completed your prescribed course of study, you will be able to:

7. Given any rational number of the form $\frac{a}{b}$, express it in decimal form and state whether it is a terminating or repeating decimal.

8. Given any rational number expressed in decimal form, write it in the form $\frac{a}{b}$ where a is a whole number and b is a natural number.

9. Given any pair of rational numbers, name the number midway between them.

10. Given any word phrase like the ones in Appendix I, translate it into an equivalent mathematical phrase.

RESOURCES

I. Reading and Problems.

1. Vanatta, Algebra One, #7,8,9,10 ____.

2. Dolciani, Modern Algebra, #7 pp. 400-402, Ex. 1-12 even p. 403: #8 pp. 400-402, Ex. 13-20 p. 403: #9, p. 398, Ex. 15-20 p. 400: #10 ___.

3. Nichols, Modern Elementary Algebra, #7 pp. 30-31, 65-77, Ex. 1-4, 5-6 EOL, p. 67-78: #8 pp. 68-70, 1-2 EOL p. 70: #9 p. 71, Ex. 1 p. 73: #10 p. 79, Ex. 1-19 pp. 79-80.

4. Dolciani, Modern School Mathematics, Algebra I, #7 pp. 422-423, Ex. 1-8 p. 426: #8 pp. 424-425, Ex. 9-19 odd p. 426: #9 p. 2, Ex. 21-22 p. 3, 19-22 p. 115: #10 p. 10, Ex. 11-23 odd p. 13, 31-38 p. 33.

5. Payne, Algebra One, #7 p. 30, Ex. ___: #8 p. 30, Ex. 64-69 p. 30: #9 ___: #10 p. 128, Ex. 1-35 odd pp. 128-130.

6. Pearson, Modern Algebra, Book One, #8 pp. 268-269, Ex. ___: #9 pp. 35-36, Ex. 1-4 p. 36: #10 p. 145, Ex. 10, 12-14 p. 147.

II. Audio

Wollensak C-3801 Open Phrase, Open Sentence

APPENDIX I

IN THIS SECTION you will learn to translate from a word phrase to a mathematical expression. This will help you when you later solve word problems. You are to fill EACH blank below with a mathematical expression. YOU ARE NOT TO DO ANY COMPUTING. (The first two problems have been completed for you as examples.)

1. The sum of 3 and the product of 2 and 6 is $3+2(6)$
   (You should not write 15 or 3 + 12, since that requires computing!)
2. Three more than the square of x is x^2+3
3. The sum of 5 and 9 is _____
4. The sum of $\frac{2}{3}$ and -6 is _____
5. The sum of 17 and x is _____
6. 5 more than 7 is _____
7. 18 increased by 12 is _____
8. x more than 10 is _____
9. 3 more than x is _____
10. The sum of $2x$ and $5 + 3x$ is _____
11. $5t$ increased by $5 - x$ is _____
12. The square of the sum of 3 and 4 is _____
13. The sum of the squares of 3 and 4 is _____
14. The square of the sum of $2x$ and $3y$ is _____
15. The sum of the squares of 5 and m is _____
16. Three times the square of x is _____
17. The square of the product of 3 and x is _____
18. The quotient of 17 divided by 6 is _____
19. The quotient of x divided by 3 is _____

20. The square of the opposite of 5 is _____

21. The opposite of the square of 5 is _____

22. The square of the opposite of x is _____

23. The opposite of the square of x is _____

24. Monday and _____ are consecutive days of the week.

25. Tuesday, _____ and Thursday are consecutive days of the week.

26. 1, 23, -62, and -14 are integers. -15, -16, -17, ____, and -19 are consecutive integers.

27. If x is an integer, then x, $x + 1$, and _____ are consecutive integers.

28. If y is an integer, then $y - 2$, $y - 1$, y, $y + 1$, _____, and $y + 3$ are consecutive integers.

29. If t is an integer, then $3t$ is an integer. Also, $3t$, $3t + 1$, _____, _____, and $3t + 4$ are consecutive integers.

30. -3, 0, 5, 7, and 212 are integers. -8, 0, 2, 16, -40, and 18 are even integers. If k is an even integer, then $k + 8$ is an _____ integer. 18, 20, _____, and 24 are consecutive even integers. If x is an even integer, x and _____ are consecutive even integers. If t is an even integer, then $t - 2$, t, $t + 2$, _____, and $t + 6$ are consecutive even integers.

31. 7 is an odd integer. 3, 5, 7, ____, and 11 are consecutive odd integers.

32. If m is an odd integer, then m and _____ are consecutive odd integers.

33. If r is an odd integer, then r, $r + 2$, _____, and $r + 6$ are consecutive odd integers.

34. The average of 6 and 4 is: _____

The average of 5, 82, 16, 93, and 74 is: _____

The average of a, b, c, and d is: _____

SELF-EVALUATION 2

7 I. Express the following fractions as decimals and state if they are repeating or terminating.

1. $\frac{4}{9}$

2. $\frac{3}{8}$

3. $\frac{2}{11}$

4. $\frac{2}{7}$

8 II. Express the following decimals as fractions.

5. .12

6. $.\overline{274}$

7. $.5\overline{3}$

8. .684

9. $.\overline{73}$

10. .82

9 III. Find the rational number midway between the following:

11. $9\frac{1}{2}$ and $11\frac{1}{3}$

12. 2.19 and 1.11

13. -3.12 and 3.76

14. $\frac{1}{6}$ and $\frac{3}{24}$

15. $\frac{3}{4}$ and $\frac{15}{16}$

10 IV. Write the mathematical phrase of each word phrase.

_____ 16. sum of 17 and x

_____ 17. 3 more than x

_____ 18. the square of the sum of 3 and x is

_____ 19. three times the square of x

_____ 20. the quotient of x divided by 2 y

_____ 21. ____ is the next consecutive odd integer after x

_____ 22. t is an integer, give the next three consecutive integers

_____ 23. number of feet in 7t yards

_____ 24. number of quarts in (a + 3t) gallons

_____ 25. worth in cents of y eight-cent stamps

APPENDIX I (Obj. 4)

Put an x by each property that holds for the given sets of numbers.
Put a circle (0) by each property that does not hold. Do not leave a blank.

PROPERTY	NATURALS	WHOLES	INTEGERS	RATIONALS	REALS
Closure for +					
Closure for x					
Commutative +					
Commutative x					
Associative +					
Associative x					
Distributive					
Add. Identity					
Mult.Identity					
Add. Inverses					
Mult.Inverses					

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Dolciani, Mary P., Wooton, William, Beckenback, Edwin F., Jurgensen, Ray C., Donnally, Alfred J., Houghton Mifflin Co., 1967.

Pearson, Helen R., Allen, Frank B., Modern Algebra, A Logical Approach, Ginn and Company, 1964.

Nichols, Eugene D., Arithmetic of Directed Numbers, Holt, Rinehart and Winston, 1962.

Wollensak teaching tape C-3801.

Equations, a game by Layman Allen.

-25 $\frac{-6}{1}$ $2^{\sqrt{2}}$ $\sqrt{2}$ $\frac{9}{7}$ 0.000076 $1\frac{5}{8}$ $\log_2 5$ .315656 π $\sqrt[3]{7}$ Sin 10° 5.78

LAP

LEARNING ACTIVITY PACKAGE

OPERATIONS OVER REAL NUMBERS

Ninety Six High School
Ninety Six, S.C. 1001

PRE ALGEBRA 93-94

REVIEWED BY _____

LAP NUMBER 49

WRITTEN BY Diane Evans

Acknowledgement

The administration and staff of Ninety Six High School gratefully acknowledges the assistance provided by the staff of Nova High School, Fort Lauderdale, Florida. We are especially indebted to Mr. Lawrence G. Insel and Mr. Laurence R. Nantuck of Nova's Math Department for permitting us to use much material developed by them, some of which has been reproduced in its original form.

RATIONALE

THE LAP's PURPOSE

In this LAP the arithmetic of real numbers is completed. It is essential to learn how to add, subtract, multiply, and divide any two real numbers to progress very far in Algebra. In the past you have studied operations over various sets. For all practical purposes the set of Reals will be the final set to be concerned with, so now is the time to survey and culminate all the ideas and properties concerning the basic operations. We do this in a mathematically formal way and call the listing of basic ideas (postulates) a field. So let's study the field of real numbers.

SECTION 1

Behavioral Objectives

After having completed your prescribed course of study, you will be able to:

1. Given any real number, name its opposite (additive inverse.

2. Given any two real numbers, name their sum.

3. Given any phrase involving subtraction, write an equivalent phrase involving addition.

4. Given any two real numbers, name their difference.

5. Given any two real numbers, name their product.

6. Given any non-zero real number, name its reciprocal (multiplicative inverse).

7. Given any phrase involving division, write an equivalent phrase involving multiplication.

8. Given any two real numbers, name their quotient.

RESOURCES I

I. Reading and Problems.
(EOL means every other letter)

1. Vanatta, Algebra One, #1 pp. 84, Ex. 7, 13 pp. 86-87: #2 pp. 90-94, Ex. 1-20 even p. 96: #3 ____: #4 pp. 96-99, Ex. 1-16 p. 99: #5 pp. 101-104, Ex. 1-26 odd p. 104: #6,#7 ____: #8 pp. 104-106, Ex. ____.

2. Dolciani, Modern Algebra, Book 1, #1 pp. 120-122, Ex. 1-10 oral p. 122: #2 pp. 124-126, Ex. 1-18 oral p. 126: #3 pp. 128-130, Ex. 1-4, 9-13 p. 131: #4 pp. 128-130, Ex. 1-35 oral p. 130: #5 pp. 133-135, Ex. 1-21 oral p. 135: #6 p. 138, Ex. 33-42 p. 141: #7 ____: #8 pp. 138-140, Ex. 1-10 p. 140, 1-18 p. 141.

3. Nichols, Modern Elementary Algebra, #1,2,3,4 pp. 85-88, 68, Ex. 6 EOL p. 68; 1, 2(any 20) p. 86-87; 1,2,3 EOL p. 89: #5 pp. 92-94, Ex. 1 p. 95: #6,7,8 pp. 96-100, Ex. 1-4 pp. 98-99, 1-4 pp. 100-101.

4. Dolciani, Modern School Mathematics, #1,2,3,4 pp. 95-98, Ex. 1-38 pp. 98-99: #5 pp. 106 - 115, Ex. 1-8 p. 109, 1-12 pp. 114-115: #6,7,8 ____.

5. Payne, Algebra One, #1,2,3,4 pp. 71-74, Ex. 1-50 p. 74: #5 pp. 75-80, Ex. 1-30 pp. 77-78: #6,7,8 pp. 82-83, Ex. 1-35 pp. 83-84.

6. Pearson, Modern Algebra, A Logical Approach, #1,2,3,4 pp. 219, 222-225, Ex. 1-4 p. 225; 3,4 p. 217: #5 pp. 227-229, Ex. 1-5 pp. 229-230: #6,7,8 pp. 230, 270-277, Ex. 1,2 pp. 272-273, 2 p. 275, 1,2,4 p. 277.

7. Nichols, Arithmetic of Directed Numbers, A Programmed Unit, #1,2,3,4 frame 230-262: #5 frames 264-367: #6,7,8 frames 368-413.

II. Audio

Wollensak Teaching Tapes C-3331 Directed Numbers: Addition

C-3332 Directed Numbers: Subtraction

C-3333 Directed Numbers: Multiplication

C-3334 Directed Numbers: Division

III. Games

The Conversion Game by D. Evans

Equations by Layman Allen

SELF-EVALUATION 1

Obj.

1 I. List the opposite (additive inverse) of the following:

_____ 1. 6

_____ 2. $\frac{1}{2}$

_____ 3. $-\sqrt{3}$

_____ 4. $|-4|$

_____ 5. $-|5|$

_____ 6. $-x$

2 II. Give the sum of the following:

_____ 7. $-\frac{1}{2} + -\frac{1}{4}$

_____ 8. $-6 + 10$

_____ 9. $-5 + -8$

_____ 10. $\frac{-5}{2} + \frac{1}{4}$

_____ 11. $+9 + -71$

3 III. Express each of the following as an equivalent phrase involving addition.

_____ 12. $4 - 3$

_____ 13. $10 - 25$

_____ 14. $(4 + 7) - (7 + 2)$

_____ 15. $(3 - 6) - (2 + 7)$

_____ 16. $(14 - 15) + [(16 + 7)] - 4$

_____ 17. $(x + y) - (x - y)$

4 IV. Give the difference of each of the following:

_____ 18. $4 - 1$

_____ 19. $-12 - 17$

_____ 20. $14 - (-3)$

_____ 21. $0 - \frac{1}{2}$

_____ 22. $-2 - (-2)$

_____ 23. $-\frac{1}{4} - (-\frac{1}{2})$

5 V. Give the product of the following:

_____ 24. $-5 \times \frac{3}{4}$

_____ 25. $-12 \times \frac{-1}{12}$

_____ 26. $0 \cdot (-\frac{1}{2})$

_____ 27. $2 \cdot -1$

_____ 28. $|4 - 6| \times (-3)$

_____ 29. $-\frac{2}{3} \times \frac{-9}{10}$

6 VI. Give the reciprocal (multiplicative inverse) of each of the following:

_____ 30. 5

_____ 31. $-\frac{1}{2}$

_____ 32. -.6

_____ 33. $\frac{2}{3}$

_____ 34. -1

7 VII. Write each of the following an an equivalent phrase involving multiplicati

_____ 35. 4 ÷ 3

_____ 36. 10 ÷ 25

_____ 37. $(4 - \frac{1}{2}) \div (\frac{-3}{4})$

_____ 38. $(2 + 3) \div (2 - 3)$

_____ 39. $(\frac{-3}{4}) \div (\frac{-3}{4})$

_____ 40. $0 \div \frac{4}{5}$

8 VIII. Find the quotient of the following:

_____ 41. $(-17) \div 2$

_____ 42. $-1000 \div -100$

SELF-EVALUATION (cont')

_____ 43. $-\frac{1}{2} \div 5$

_____ 44. $\frac{3}{10} + \frac{-17}{2}$

_____ 45. $-.25 + \frac{-2}{5}$

IF YOU HAVE SATISFACTORILY COMPLETED YOUR WORK, YOU MAY TAKE YOUR PROGRESS TEST. CONSULT YOUR TEACHER FIRST.

SECTION 2

Behavioral Objectives

After having completed your prescribed course of study, you will be able to:

9. Write the simplest name for any given phrase involving a combination of addition, subtraction, multiplication, and/or division.

10. Given any subset of the real numbers, determine which of the field properties are true in the set, by answering questions or by completing a chart like the one in Appendix I.

11. Given an open sentence with at least one unknown and a universal set which is a subset of the reals, determine the set of all replacements for the unknown(s) that will make that sentence true. (That is the solution set for the sentences.) You will determine the solution set by observation only.

12. Given a word phrase like the ones in Appendix II, write it as an equivalent mathematical phrase.

RESOURCES 2

I. Reading and Problems

1. Vanatta, Algebra One, #9 ____, Ex. 9-25 p. 107: #10 ____:#11,#12, ____.

2. Nichols, Modern Elementary Algebra, #9 pp. 31-32, 101-102, Ex. 1 p. 103: #10 pp. 104-105, Ex. 1-4 pp. 105-106: #11 ____, Ex. 5-7 p. 90, 2 p. 95, 5 p. 101, 2 EOL p. 103: #12 p. 107, Ex. 1-22 p. 108.

3. Dolciani, Modern School Mathematics, #9 ____, Ex. 1-20 p. 99, 23-30 p. 115: #10 ____: #11 ____, Ex. 9-12 p. 109: #12 ____.

4. Payne, Algebra One, #9 ____, Ex. 45-48 and 55-66 p. 84: #10 ____: #11 ____, Ex. 1-12 p. 72, 36-44 and 49-54 p. 84: #12 ____.

5. Pearson, Modern Algebra, L. A., #9 ___, Ex. 7,12,15 p. 265, 9 p.267: #10 pp. 209-212, Ex. ___: #11 ___, Ex. 12, 13 p. 226, 7 p. 230: #12 ___, Ex. 6-11 p. 226(Write an equivalent math phrase.)

II. Audio

Wollensak Teaching Tape - C-3458 The Real Number System
C-3453 The Commutative Property
C-3454 The Associative Property
C-3456 The Closure Property
C-3457 The Inverse Elements
C-3801 Open Phrase, Open Sentence
C-3459 The Identity Element

III. Games

Propo (a game on the Real Number system and its properties)

APPENDIX I (Obj. 10)

Put an x by each property that holds for the given sets of numbers.
Put a circle (0) by each property that does not hold. Do not leave a blank.

PROPERTY	NATURALS	WHOLES	INTEGERS	RATIONALS	REALS
Closure for +					
Closure for x					
Commucative +					
Commutative x					
Associative +					
Associative x					
Distributive					
Add. Identity					
Mult.Identity					
Add. Inverses					
Mult.Inverses					

APPENDIX II (Goal 12)

1. 5 less than 7 is _____.
2. 7 less than 5 is _____.
3. 3 less than x is _____.
4. x less than 3 is _____.
5. 10 decreased by 2 is _____.
6. 15 decreased by 5 is _____.
7. 7 decreased by x is _____.
8. 3t decreased by 5n is _____.
9. 7r less than 10t is _____.
10. 4 less than t + 5a is _____.
11. The absolute value of the sum of 3x and 2y is _____.
12. The sum of the absolute values of 3x and 3y is _____.
13. 5 times the sum of 6 and 2 is _____.
14. The product of 8 and the sum of 2 and t is _____.
15. The product of the sum 5 and 2t and the sum of 9 and 7t is _____.
16. The quotient of 8x divided by the sum of t and 3 is _____.
17. The quotient of the sum of 3 and p divided by the product of 3 and p is _____.
18. The reciprocal of 7 is _____.
19. The reciprocal of m is _____.
20. The sum of the reciprocals of 7 and m is _____.
21. The reciprocal of the sum of 7 and m is _____.
22. 7 nickels are worth _____ cents.
23. 42 nickels are worth _____ cents.
24. x nickels are worth _____ cents.
25. c + 2 nickels are worth _____ cents.
26. 3x nickels are worth _____ cents.

SELF-EVALUATION 2

Obj.

9 I. Perform the indicated operation (remember the order of operation from a previous LAP.)

_____ 1. $-4 + 7 - (-3)$

_____ 2. $2 - (-4) + 3$

_____ 3. $-3 \times 4 - 6$

_____ 4. $17 \times (-6) + 17 \times (-4)$

_____ 5. $9 - 12 + 7 + 6 - 4$

_____ 6. $2 \times (-3) \div (-2) \times \frac{1}{2} + 12 \div (-6)$

10 II. True or False.

_____ 7. The set of natural numbers form a field with operations addition and multiplication.

_____ 8. Every element in a field has a multiplicative inverse.

_____ 9. In a field addition and multiplication are associative.

_____ 10. The integers do not have multiplicative inverses.

_____ 11. The natural numbers have all the properties except additive inverses.

_____ 12. The irrational numbers do not have an additive identity.

_____ 13. The rational numbers form a field.

_____ 14. Every integer has an additive inverse.

_____ 15. Addition is associative.

_____ 16. Subtraction is commutative.

11 III. In the following write the number(s) that make each statement true.

_____ 17. $3 \cdot 7 + -4 = x$

_____ 18. $P - 6 = -12$

_____ 19. $\frac{1 + x}{2} = -2$

_____ 20. $3x + 2 = -13$

_____ 21. $x \cdot -4 = -24$

Given the following open sentences and universal sets, determine the solution set of each sentence.

_____ 22. $2x < -6$ $\{...-4,-3,-2,-1,0\}$

_____ 23. $-6 + n = 20$ $\{14,-14,26,-26\}$

_____ 24. $x > -5$ $\{-6,-4,-2,-1,0\}$

_____ 25. $x \leq |-4|$ $\{-4,-3,-2,-1,0,1,2,3,4\}$

_____ 26. $x > |-2|$ $\{-2,-1,0,1,2\}$

If the universal set is the set of real numbers, are the following TRUE or FALSE?

_____ 27. $\forall x < 0$ and $\forall y > 0$ then $x - y < 0$

_____ 28. $\forall x < 0$ and $\forall y < 0$ then $x \cdot y < 0$

_____ 29. $\forall x$ then $x \cdot 1 = 1$

12 IV. Write the following as a mathematical phrase.

_____ 30. The reciprocal of 2m.

_____ 31. The sum of the reciprocals of x and y.

_____ 32. If Dave is 2n years older than Susie and Susie is $3x + 1$ years old, how old is Dave?

_____ 33. The reciprocal of the sum of x and y.

_____ 34. If Ed was $x + 1$ years old 3 years ago, how old is he now?

_____ 35. 12 increased by $n - 4$.

If you have mastered all the behavioral objectives for section II, take your LAP TEST. CONSULT YOUR TEACHER FIRST.

ADVANCE STUDY

I. Answer all the following:

1. If 15 mice were split into x groups for diet-growth experiments, each group would have _____ mice.

2. If you start out with 20 amoebas at the death rate of 2/hour, after x hours you would have _____ amoebas left.

3. If you start out with 7 pair of mice and each female has y mice per litter, after each female has given birth once, you will have a total of _____ mice.

4. Bacteria media costs 25¢ a gram. If you get 15 grams and then find you need x more grams, what is the total cost of the media?

5. Every moving object has momentum. Momentum (P) is defined as the product of the mass (m) and velocity (v). Write the formula for momentum.

6. An object moving with uniform speed (v) for a period of time (t) will be displaced a distance (D). If the distance is equal to the product of the velocity and time, write this as a formula.

7. The density (D) of a material is its mass (m) divided by its volume (v). Write the formula for density.

8. For an ideal gas, the product of pressure (P) and volume (V) is equivalent to the product of the number of moles (n), the temperature (T) and the gas constant R. Write this as a formula.

ADVANCED STUDY (cont')

II. Make a chart or bulletin board using Venn Diagrams showing the real number system and all of its subsets.

III. Determine if the set of irrational numbers is a field by listing the field properties, giving an example of each using irrational numbers, and explaining whether or not each property applies.

IV. Prove any two of the following:

a. If $ac = bc$ and $c \neq 0$, then $a = b$.

b. If $c + a = c + b$, then $a = b$.

c. If $a = bc$ and $b \neq 0$, then $c = a \cdot \frac{1}{b}$

d. If $b = d$ and $a = b = c - d$, then $a = c$.

V. Complete our number system by studying the complex numbers. Read pages 474-476 in Vanatta and work exercises 1-24 on pages 476-477.

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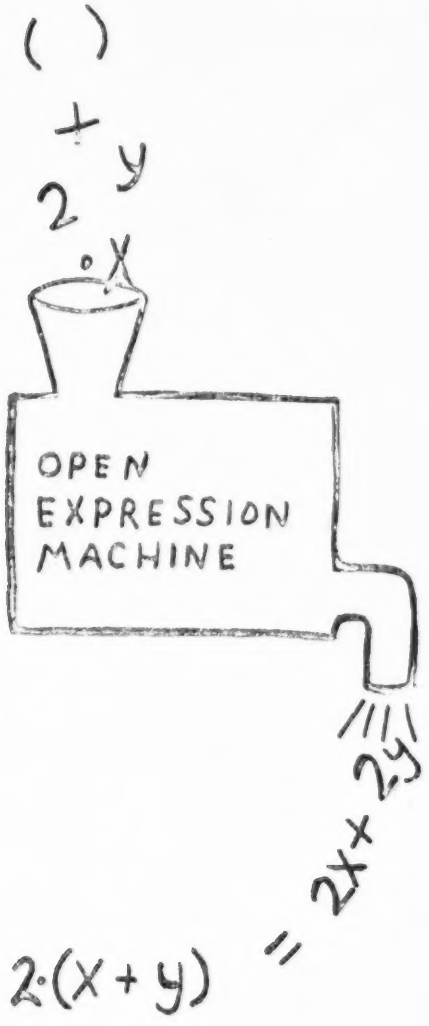
LAP

LEARNING ACTIVITY PACKAGE



OPEN

EXPRESSIONS

Ninety Six High School - Ninety Six, S.C.

PRE ALGEBRA 93-94

WED BY

LAP NUMBER 810

WRITTEN BY Diane Evans

Acknowledgement

The administration and staff of Ninety Six High School gratefully acknowledges the assistance provided by the staff of Nova High School, Fort Lauderdale, Florida. We are especially indebted to Mr. Lawrence G. Insel and Mr. Laurence R. Wantuck of Nova's Math Department for permitting us to use much material developed by them, some of which has been reproduced in its original form.

RATIONALE (The LAP's Purpose)

One of the most important concepts in the study of Algebra is that of a variable. In this LAP you will study terms and expressions, most of which contain one or more variables. Using many of the previously introduced properties and definitions, you will prove and learn to apply new theorems, listed on the next page, which are concerned with equivalent expressions.

You will develop the ability to judge whether two given expressions are equivalent. This skill is necessary in solving equations. While you are developing this skill, you will begin to learn how to prove theorems.

PROPERTIES AND THEOREMS

For every number x, y, and z, the following apply:

Distributive property of multiplication over addition

$$x(y + z) = xy + xz$$

$$xy + xz = x(y + z)$$

$$(y + z)x = yx + zx$$

Distributive property multiplication over subtraction

$$x(y - z) = xy - xz$$

$$(y - z)x = yx - zx$$

$$xy - xz = x(y - z)$$

Multiplication by -1 $\quad x(-1) = -x$

Division by -1 $\quad \frac{x}{-1} = -x$

Opposite of x - y $\quad -(x - y) = y - x$

Opposite of x + y $\quad -(x + y) = -x + -y$

$(-x)y = -(xy)$

Some additional theorems to be covered in this LAP

$-(-x) = x$

$-(-x)y = xy$

$(-x)(-y) = xy$

Behavioral Objectives

At the completion of your prescirbed course of study, you will be able to:

1. Given any polynomial, classify it as:

   a. monomial
   b. binomial
   c. trinomial

2. Given an open expression and replacements for the variables, compute the value of the expression.

3. Given a pair of expressions, determine whether or not they are equivalent.

4. Given a proof that two expressions are equivalent, state reasons to justify each step of that proof.

5. Using the appropriate properties, definities, and theorems, write equivalent expressions for any given expression.

6. Given a word phrase like the ones in the Appendix, change it to an equivalent mathematical phrase. (The Appendix will be turned in to your teacher.)

COMMENT: You probably notice that there is not an objective asking you to perform proofs. We would like to emphasize that these objectives are just the minimum amount we expect you to perform. We encourage all students to attempt some proofs.

RESOURCES

Goals 1, 2

Vanatta, #1 read p. 67-71, Ex. 4 p. 71: #2 read p. 72, Ex. 1-27 p. 73.
Dolciani, Modern Algebra, Bk. 1, #1 ___: #2 read pp. 36-37, Ex. 1-9 written p. 37, 37-46 p. 43.
Nichols, #1 and 2, read pp. 119-122, Ex. 1, 2 pp. 121-124.
Wooton, #1 and 2, read pp. 52-54, Ex. 1-19 oral p. 54.
Payne, #1 and 2, read pp. 307-309, Ex. 1-10, and 18-27, pp 309-310.
Pearson, #1 and 2, read pp. 240-241, Ex. 1,2 p. 241, and 3 p. 243.

Goals 3, 4

Nichols, read pp. 124-132, Ex. pp. 124 (fill reasons in for theorem 1)
pp. 127-128 no. 6, 7
pp. 130-131, nos. 1-12
pp. 131 (fill reasons in for theorem 7)
pp. 132, nos. 3, 4; proofs pp. 127, nos. 1-5; nos. 1, 2, 5, 6
Payne, read pp. 86-87, Ex. 1-19 pp. 87-88.

RESOURCES (cont')

Goal 5

Vanatta, read pp. 67-71, Ex. 5 p. 71, nos. 1-20 even p. 125, no. 3 p. 75.

Dolciani, pp. ___, Ex. 1-30 even p. 79.

Nichols, read pp. 133-137, Ex. 2-6 pp. 135-136.

Wooton, read pp. 55-57, Ex. 7-24 p. 58, 1-28 p. 59.

Payne, read pp. 88-90, Ex. 1-20 p. 90, 39-43 p. 92.

Pearson, read pp. 242-243, Ex. 1-27 p. 242, nos. 1, 2 p. 243.

Goal 6

Vanatta, pp. ___, Ex. 1-6 p. 74, 1 p. 75, 1 p. 77.

Dolciani, read pp. ___, Ex. 1-24 p. 42.

Nichols, read p. 137, Ex. 1-13 pp. 138-139.

Wooton, read pp. ___, Ex. 29-32 p. 59.

Wollensak tape C-3801 Open Phrase.

* Appendix

* required

APPENDIX
(to be turned in to the teacher)

1. 3 dimes are worth _____ cents.
2. x dimes are worth _____ cents.
3. 7x dimes are worth _____ cents.
4. $\frac{x}{2}$ dimes are worth _____ cents.
5. x - 4 dimes are worth _____ cents.
6. 7 3-cent stamps are worth _____ cents.
7. k + 4 3-cent stamps are worth _____ cents.
8. 5t 7-cent stamps are worth _____ cents.
9. If I have 3 nickels and 4 dimes and 2 quarters, then I have _____ coins worth _____ cents.
10. If I have 4 nickels and x dimes and 3 quarters, then I have _____ coins worth _____ cents.
11. If I have x nickels and 3x dimes and x + 2 quarters, then I _____ have coins worth _____ cents.
12. Al is 12 years old. 5 years ago he was _____ years old and 8 years from now he will be _____ years old. 3 times his present age is _____. Bill is 4 years younger than Al. Bill is _____ years old.
13. Ed is x years old. 3 years from now he will be _____ years old and 2 years ago he was _____ years old. Dave is 4 times as old as Ed is now. Dave is _____ years old. 2 years ago he was _____ years old. Hal is 2 years younger than Ed. Hal is _____ years old. In 5 years he will be _____ years old. Sam is 6 years older than Dave. Sam is _____ years old.

SELF-EVALUATION

OBJ. 1

Classify the following as monomials, binomials, or trinomials.

_____ 1. x^5

_____ 2. $x^4 + 6x^2$

_____ 3. $5 - 4x + 2y$

_____ 4. $456x^4y^5$

_____ 5. $4x^3y^2r^5 + 6$

OBJ. 2

Determine the value of each expression if the replacement for x is 5 and the replacement for y is -3.

_____ 6. $5x^2$

_____ 7. $x + y$

_____ 8. $4x^2 + 3y$

_____ 9. $\frac{x^2 + y}{x}$

_____ 10. $\frac{x^3 - y^3}{y}$

OBJ. 3

Given the following pair of expressions, are they equivalent? Write Yes or No.

_____ 11. $x(-y)$ and $(-x)y$

_____ 12. $4 - x$ and $x - 4$

_____ 13. $-(a - b)$ and $b - a$

_____ 14. $-(a + b)$ and $b + a$

_____ 15. $(a + b)^3$ and $a^3 + b^3$

OBJ. 4

Given the following proofs state the reason which justify each step.

Prove: $(-x)y = -(xy)$ Reason

$(-x)y = [x\ (-1)]\ y$ 16.

$= x\ [(-1)y]$ 17.

$= x\ [y\ (-1)]$ 18.

$= (xy)\ \ (-1)$ 19.

$= -(xy)$ 20.

Prove: $-(K + L) = -K + -L$ Reason

Proof: $-(K + L) = (K + L)(-1)$ 21.

$= (-1)(K + L)$ 22.

$= (-1)\ K + (-1)L$ 23.

$= K(-1) + L(-1)$ 24.

$= -K + -L$ 25.

OBJ. 5

Change each of the following expressions to equivalent expressions with the least number of terms (simplify)

_____ 26. $4x + 3x$

_____ 27. $7y - 8y$

_____ 28. $-5\ (x - 4y) + 5\ (x - y)$

_____ 29. $C - 3d - C + 4$

_____ 30. $-\frac{3}{4}x - 7 + \frac{2}{5}y + \frac{5}{8}x + 5$

_____ 31. $4\ (xr - 5x) - 2(2xr - 10x)$

____ 32. $5(xy + 3c) - 2(4c + 2xy)$

____ 33. $-6x + 5y - (3x + 2y)$

____ 34. $-(3x + 4y) - (6x + 2y)$

____ 35. $-3(4x + 7y) + 2(7x - 9y)$

____ 36. $6(x + 4) - 3(x + 2) - (x -$

OBJ. 6

Write the equivalent mathematical phrases for the following word phrases.

_____ 37. the sum of x and twice y

_____ 38. the difference of 4t and 6 multiplied by the sum of 4t and 6

_____ 39. the number of quarts in 3y pints

_____ 40. Worth in cents of (4y - 1) nickels

_____ 41. John was 5r + 6 years old five years ago, how old will he be nine years from now?

If you have successfully completed your work, you may take the LAP TEST.

ADVANCED STUDY

I. Complete the following:

LINEAR

METER	DECIMETER	CENTIMETER	MILLIMETER
1	10	100	1000
x		100x	
5			
x + 5			
7x			

MASS

Kilogram	Gram
1	1000
2	
3	
5	
2.5	
x	
x + 2	
6x	
$\frac{x}{1000}$	

VOLUMETRIC

Liter	Milliliter (c.c.)
1	
2	
3	
6.1	
x	
x - 2	
12x	
$\frac{x}{100}$	

1. 2 meters, 5 centimeters is_____ millimeters.
2. 3 meters, x centimeters is_____ millimeters.
3. y meters, x centimeters is_____ millimeters.
4. 3 + x meters, 2x centimeters is_____ millimeters.

ADVANCED STUDY (cont')

II. Prepare a chart using a Venn diagram showing the relationships among polynomials, monomials, binomials, and trinomials.

III. Dolciani, Modern Algebra, Bk. 1, work any ten problems from 1-15 on pages 43-44.

IV. Nichols, page 132, number 7.

V. Nichols, page 141, number 3.

REFERENCES

Nichols (abbreviation)

Nichols, Eugene, D., Modern Elementary Algebra, Holt, Rinehart and Winston, Inc., 1965.

Pearson (abbreviation)

Pearson, Helen R., and Allen, Frank B., Modern Algebra: A Logical Approach, Ginn and Company, 1964.

Payne (abbreviation)

Payne, Joseph N., Zamboni, Floyd F., Lankford, Jr., Francis, Algebra One, Harcourt, Brace and World, Inc., 1969.

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Dolciani, Mary P., Wooton, William, Beckenback, Edwin Fr., Jurgensen, Ray C., Donnelly, Alfred J., Modern School Mathematics, Algebra 1.

Dolciani (abbreviation)

Dolciani, Mary P., Berman, Simon L., Freilich, Julius, Modern Algebra, Book 1, Houghton Mifflin Co., 1965.

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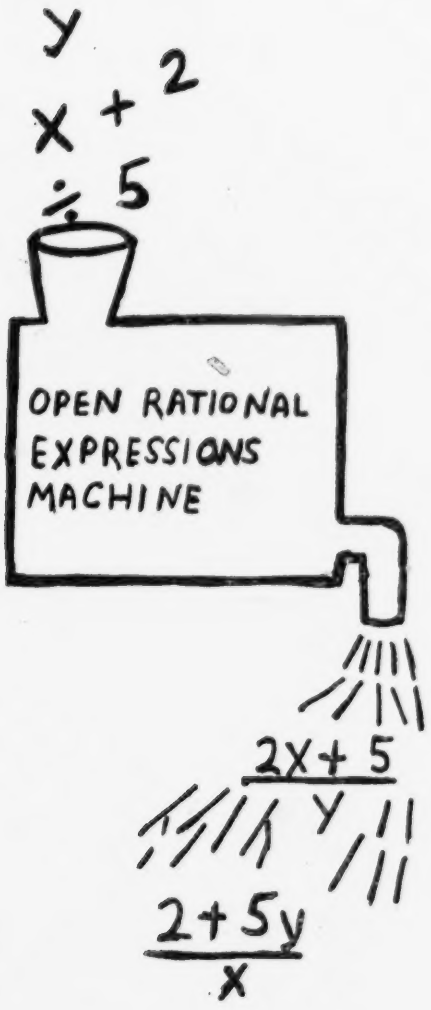
Vanatta, Glen D., Goodwin, A. Wilson, Algebra One, Charles E. Merrill Publishing Co., 1966.

Wollensak teaching tape C-3801 - Open Phrase .

LEARNING
ACTIVITY
PACKAGE

OPEN RATIONAL EXPRESSIONS



Ninety Six High School
Ninety Six, S.C.

PRE Algebra 93-94

IEWED BY J.R.

LAP NUMBER 811

WRITTEN BY Diane Evans

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RATIONALE (The LAP's Purpose)

In LAP 5 you learned some new properties and definitions for operations with open expressions. Operations on these open expressions is analogous to operations on natural numbers. Now, in this LAP, you will combine your knowledge of variables with basic properties of numbers and operations to justify theorems for operations with open rational expressions. These are expressions which serve as a basis for computation with rational numbers.

As in LAP 5, in this LAP, you will apply theorems and properties of numbers to improve your ability to determine when open rational expressions are equivalent. The skills you develop here are important basis for your future work with equations in two or more variables.

On the following page is a list of theorems which you will learn and apply in this LAP.

THEOREMS TO BE DEVELOPED IN THIS LAP

$\Psi_x \Psi_y \neq o \Psi_z \quad \frac{xz}{y} = (\frac{x}{y})(z)$

$\Psi_x \Psi_y \neq o \Psi_z \quad \frac{xz}{y} = (x)(\frac{z}{y})$

$\Psi_x \Psi_y \neq o \Psi_z \quad \frac{xz}{y} = (\frac{1}{y})(xz)$

$\Psi_{x\neq o} \Psi_{y\neq o} \quad (\frac{1}{x})(\frac{1}{y}) = \frac{1}{xy}$

$\Psi_x \Psi_{y\neq o} \Psi_r \Psi_{s\neq o} \quad (\frac{x}{y})(\frac{r}{s}) = \frac{xr}{ys}$

(Multiplicative Identity Theorem) $\Psi_x \Psi_{y\neq o} \Psi_{r\neq o} \quad \frac{xr}{yr} = \frac{x}{y}$

(Addition Theorem) $\Psi_x \Psi_{y\neq o} \Psi_r \Psi_{s\neq o} \quad \frac{x}{y} + \frac{r}{s} = \frac{xs + ry}{ys}$

(Subtraction Theorem) $\Psi_x \Psi_{y\neq o} \Psi_r \Psi_{s\neq o} \quad \frac{x}{y} - \frac{r}{s} = \frac{xs - ry}{ys}$

(Division Theorem) $\Psi_x \Psi_{y\neq o} \Psi_{r\neq o} \Psi_{s\neq o} \quad \frac{\frac{x}{y}}{\frac{r}{s}} = \frac{x}{y} \cdot \frac{s}{r}$

$\Psi_x \Psi_{y\neq o} \quad \frac{-x}{y} = -\frac{x}{y}$

$\Psi_x \Psi_{y\neq o} \quad \frac{x}{-y} = -\frac{x}{y}$

$\Psi_x \Psi_{y\neq o} \quad \frac{-x}{-y} = \frac{x}{y}$

$\Psi_x \Psi_{y\neq o} \quad -(\frac{x}{-y}) = \frac{x}{y}$

$\Psi_x \Psi_{y\neq o} \quad -(\frac{-x}{y}) = \frac{x}{y}$

$\Psi_x \Psi_y \Psi_{r\neq o} \Psi_{s\neq o} \quad \frac{x(-y)}{r(-s)} = \frac{xy}{rs}$

OPEN RATIONAL EXPRESSIONS

BEHAVIORAL OBJECTIVES

SECTION 1

1. Given a proof involving multiplication of rational expressions, state the reason for each step in the proof.
2. Given a pair of rational expressions, write a single equivalent expression that names their product.
3. Given a rational expression, use the multiplicative identity theorem to write a single equivalent expression where the numerator and denominator have no common factors.
4. Given a proof involving addition of rational expressions, state the reason for each step in the proof.
5. Given a pair of rational expressions, write a single equivalent expression that names their sum.
6. Given a proof involving subtraction of rational expressions, state the reason for each step in the proof.
7. Given a pair of rational expressions, write a single equivalent expression that names their difference.

SECTION 2

8. Given a proof involving division of rational expressions, state the reason for each step in the proof.
9. Given a pair of rational expressions, write a single equivalent expression that names their quotient.
10. Given a proof involving the additive inverse of an expression, state the reason for each step in the proof.
11. Given a pair of rational expressions which involves the additive inverse of an expression, write a single expression equivalent to it.
12. Given any complex rational expression, use the appropriate properties, theorems and definitions to write a single expression equivalent to it.
13. Given a word phrase, change it to an equivalent mathematical phrase.

SECTION 1

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

1. Given a proof involving multiplication of rational expressions, state the reason for each step in the proof.

2. Given a pair of rational expressions, write a single equivalent expression that names their product.

3. Given a rational expression, use the multiplicative identity theorem to write a single equivalent expression where the numerator and denominator have no common factors.

4. Given a proof involving addition of rational expressions, state the reason for each step in the proof.

5. Given a pair of rational expressions, write a single equivalent expression that names their sum.

6. Given a proof involving subtraction of rational expressions, state the reason for each step in the proof.

7. Given a pair of rational expressions, write a single equivalent expression that names their difference.

NOTE: You may notice there are no objectives requiring that you perform proofs. We would like to emphasize that these objectives are just a minimum amount we expect you to perform. We encourage all students to do some proofs.

RESOURCES

Obj. 1

Nichols, read pp. 146-150, study Theorem 1 p. 147, Ex. Prove Theorems 2,3: study pages 148-149, Ex. Give reasons for Theorem 4, Prove Theorem 5 page 150.

Obj. 2

Vanatta, read pages 320-321, Ex. 1-5, 7-10 page 322.

Dolciani, MA, read pages 292, Ex. 1-20 even (oral) page 293.

Nichols, read pages 146-150, Ex. 1,2 p. 148, 3a,c,d,f,g,i page 148; 3a,c,e,g,i, 4 page 150.

Payne, read pp. 386-394, Ex. 1-9, 11-31 odd, pages 387-388; 1-11,15, 17,19 pages 394-395.

Wooton, MSM, read pp. 320-323, 328-330, Ex. 1-10(written) pp. 323-324; 1-10 p. 330; 1-7,9,16 p. 331.

Pearson, read pp. 397-400, 401-402; Ex. 1,2,4abdfhkmq age 398; ladfhkloq page 400; 1-4, 7, 8, 10, 12, 16, 20-22 pages 402-403.

Obj. 3

Nichols, read page 151, Ex. lacdeg, 2 page 152.

Payne, read pp. 405-408, Ex. 1-3, 5-21 odd, 22, 25 page 408.

Obj. 4

Nichols, read pp. 152-153, Ex. complete proofs of addition theorems on page 152.

Obj. 5

Vanatta, read pp. 236-330, 331-332, 333-335, Ex. 1,2,3,6,8 page 328; 1,4,5,7,9,10,11,12,16 page 330; 1,3,4,6,8,9,10,11,13,15 page 333; 1,3,5,6,8,9,10,12,14,15,23,24 page 335.

Dolciani, MA, read pp. 298-300, Ex. 1-14 even pages 298-299; 1-16 even, 20 p. 301.

Nichols, read pp. 152-153, Ex. la,c,e,g,i,j,l page 153.

Wooton, MSM, read pp. 332-336, Ex. 7,12 page 334; 1,3,5,7,10,11,13,15 pages 337-338.

Payne, read pp. 398-403, Ex. 1,4,5,7,12,13,15,16,18,20,23,27,28,32,35, 36,39,48,50,54,60,62,64,66 pages 403-405.

Pearson, read pp. 403-405, Ex. 2,4,7,8,12 page 405.

Obj.6

Nichols, read pp. 152-153, Ex. 3 page 153.

Obj.7

Vanatta, read pp. 326-335, Ex. 4,5,7,9-12, page 328; 2,3,6,8,13,14,15, 17,18 page 330; 2,5,7,12,14 page 333; 2,4,7,11,13,16,18,20,21,25 page 335.

Dolciani, MA, read pp. 298-300, Ex. 1-14 odd pages 298-299; 1-16 odd, 19, 21, 22, page 301.

Wooton, MSM, read pp. 332-336, Ex. 1-6.9,11 page 334; 4,8,9,12,14, 16,17,18 pages 337-338.

Payne, read pp. 398-403, Ex. 8,9,11,21,22,24,29,30,31,33,38,41,44, 56,60,61,63,65 pages 403-405.

Pearson, read pp. 403-405, Ex. 3,5,6,9-11 page 405.

SELF-EVALUATION 1

Obj.

1 I. Complete the following proofs by writing the correct reason in the blank space provided.

(1) Prove: $\Psi_x \Psi_y \neq o \Psi_r \Psi_s \neq o \; (\frac{x}{y})(\frac{r}{s}) = \frac{xr}{ys}$

Proof:

Statements	Reasons
a. $(\frac{x}{y})(\frac{r}{s}) = \left[x(\frac{1}{y})\right]\left[r(\frac{1}{s})\right]$	_____
b. $= x\left[(\frac{1}{y})r\right](\frac{1}{s})$	_____
c. $= x\left[r(\frac{1}{y})\right](\frac{1}{s})$	_____
d. $= (xr)(\frac{1}{y} \cdot \frac{1}{s})$	_____
e. $= (xr)(\frac{1}{ys})$	_____
f. $= \frac{xr}{ys}$	_____

4 (2) Prove: $\Psi_x \Psi_r \Psi_y \neq o \; \Psi_s \neq o \; \frac{x}{y} + \frac{r}{s} = \frac{xs + ry}{ys}$

Proof:

Statements	Reasons
a. $\frac{x}{y} + \frac{r}{s} = \frac{xs}{ys} + \frac{ry}{sy}$	_____
b. $= \frac{xs}{ys} + \frac{ry}{ys}$	_____
c. $= (xs)(\frac{1}{ys}) + (ry)(\frac{1}{ys})$	_____
d. $= (xs + ry)(\frac{1}{ys})$	_____
e. $= \frac{xs + ry}{ys}$	_____

6 (3) Prove: $\Psi_x \Psi_y \neq o \; \Psi_r \Psi_s \neq o \; \frac{x}{y} - \frac{r}{s} = \frac{xs - ry}{ys}$

Proof:

Statements	Reasons
a. $\frac{x}{y} - \frac{r}{s} = \frac{xs}{ys} - \frac{ry}{sy}$	_____
b. $= \frac{xs}{ys} + (-\frac{rs}{sy})$	_____
c. $= \frac{xs}{ys} + -\frac{(rs)}{sy}$	_____
d. $= \frac{xs}{ys} + -\frac{(rs)}{ys}$	_____
e. $= \frac{xs + -(rs)}{ys}$	_____
f. $= \frac{xs - rs}{ys}$	_____

Obj.

II. For each of the following expressions, write its equivalent in a single expression.

2 (1) $\frac{1}{3} \cdot \frac{2}{7}$ (2) $-\frac{1}{2} \cdot \frac{5}{3}$ (3) $\frac{-2}{-3} \cdot \frac{5}{7}$

(4) $\frac{6}{y^2} \cdot \frac{y}{2}$ (5) $\frac{a-b}{2} \cdot \frac{8}{a-b}$ (6) $\frac{5ab^2}{21c^2} \cdot \frac{3bc^2}{8a^2} \cdot \frac{7a^2}{30c^4}$

3 (7) $\frac{2-a}{a} \cdot \frac{a}{3-a}$ (8) $\frac{7}{6} \cdot \frac{-6}{6}$ (9) $\frac{1}{xy} \cdot \frac{x(y-1)}{2}$

5 (10) $\frac{2}{3} + \frac{1}{7}$ (11) $\frac{a}{b} + \frac{x}{y}$ (12) $\frac{-3}{4} + \frac{-1}{2}$

(13) $\frac{a}{7} + \frac{a}{3}$ (14) $\frac{2x}{5y} + \frac{x}{2y}$ (15) $\frac{3}{x} + \frac{4}{y}$

7 (16) $\frac{5}{8} - \frac{2}{3}$ (17) $\frac{1}{2} - \frac{2a-1}{a}$ (18) $\frac{x}{y} - \frac{-2}{3}$

(19) $\frac{5b}{3y} - \frac{3b}{4y}$ (20) $\frac{2}{x} - 5$ (21) $\frac{-3}{x} - \frac{-2}{y}$

III. True or False.

2 _____ 22. $\frac{3}{5} \cdot \frac{4}{5} = \frac{12}{5}$

_____ 23. $(-\frac{3}{4})(-2) = \frac{(-3)(-2)}{4}$

_____ 24. $\frac{3}{x} \cdot \frac{2}{y} = \frac{3y}{2x}$

_____ 25. $(-x) \cdot y \cdot \frac{1}{(-y)} = x$

_____ 26. $\frac{KL}{t} = \frac{1}{t}(LK)$

3 _____ 27. $\frac{x-3}{x-2} = \frac{3}{2}$

_____ 28. $\frac{2}{-3} \cdot \frac{-3}{5} = \frac{2}{5}$

_____ 29. $\frac{3m}{7+m} = \frac{3}{7}$

125

_____ 30. $\frac{4y}{5y} = \frac{4}{5}$

_____ 31. $\frac{(-7)x2}{5x(-7)} = \frac{2}{5}$

_____ 32. $\frac{3-x}{4-x} = \frac{3}{4}$

5 _____ 33. $\frac{2}{4} + \frac{5}{3} = \frac{7}{7} = 1$

_____ 34. $\frac{3}{2x} + \frac{4}{3x} = \frac{17}{6x}$

_____ 35. $\frac{2}{x} + \frac{1}{y} = \frac{3}{xy}$

_____ 36. $\frac{x}{3} + \frac{k}{4} = \frac{4x + 3k}{12}$

_____ 37. $\frac{-2}{3} + \frac{4}{7} = \frac{2}{21}$

_____ 38. $\frac{-3k}{2} + \frac{-k}{4} = \frac{-7k}{4}$

7 _____ 39. $\frac{1}{y} - 2 = \frac{1 - 2}{y} = \frac{-1}{y}$

_____ 40. $\frac{a}{2} - \frac{b}{8} = \frac{a-b}{6}$

_____ 41. $\frac{-x}{3} - \frac{y}{6} = \frac{-2x - y}{6}$

_____ 42. $\frac{3a}{2b} - \frac{5b}{6a} = \frac{9a^2-5b^2}{6ab}$

_____ 43. $\frac{a}{2} - \frac{-a}{3} = \frac{5a}{6}$

_____ 44. $\frac{-2}{3} - \frac{4}{2} = \frac{-6}{5}$

_____ 45. $\frac{3}{4} - \frac{-2}{3} = \frac{17}{12}$

IF YOU HAVE SATISFACTORILY COMPLETED YOUR WORK, YOU MAY TAKE YOUR PROGRESS TEST. CONSULT YOUR TEACHER FIRST.

SECTION 2

Behavioral Objectives

At the completion of your prescribed course of study, you will be able to:

8. Given a proof involving division of rational expressions, state the reason for each step in the proof.

9. Given a pair of rational expressions, write a single equivalent expression that names their quotient.

10. Given a proof involving the additive inverse of an expression, state the reason for each step in the proof.

11 Given a pair of rational expressions which involves the additive inverse of an expression, wri[illegible] single expression equivalent to it.

12. Given any complex rational exp[illegible]sion, use the appropriate properties, theorems and definitions to write a single expression equivalent to it.

13. Given a word phrase, change it to an equivalent mathematical phrase.

RESOURCES

Obj. 8

Nichols, read pp. 154-155, study Division theorem page 154 and theorems 10,11, Ex. complete theorem 12.

Obj. 9

Vanatta, read pp. 323-324, Ex. 1-15 pp. 324-325.

Dolciani, MA, read p. 295, Ex. 1-12 p. 295; 1-4 pp. 296-297.

Nichols, read pp. 154-155, Ex. 1 a,b,d,e,g,i,k,l, and 2 a,b,e,f,h, i,j,l,m pages 156-160.

Payne, read pp. 396-397, Ex. 1-22 even page 397.

Pearson, read pp. 401-402, Ex. 5,9,13,14,15,17,18 pages 402-403.

Obj.10

Nichols, read pp. 154-155, Ex. 3,4,5, pp. 156-160.

Obj. 11

Nichols, read pp. 154-155, Ex. 7 a,c,d,f,g,j,l; 8 a,b,c,d,g,h,i,l, m,o,p,q,s,u,v; 10 a,c,d,g,j,k,o,q, pages 158-160.

Obj. 12

Vanatta, read pp. 341-342, Ex. 1-7, 10, 12, 13 page 343.

Dolciani, MA, read p. 304, ex. 1-8, 11, 12, 15, 16, 23, page 305.

Nichols, read pp. 161-163, Ex. 9 a,b,c,e,f,g,h,i,j,k,l,m,n,q,r,t; 11 a,c,d,e,f,g,n,p,q,r,s,i,l,m; 12 a,c,e,f, pages 159-161. Ex. 1 a,c,f,h,j,m,n,p; 2 a,c,d,f,g,h,k,l,m,o,p,q,s,t,v,x,y,z, a',b',c',e',g',i',j',k',l', pages 163-164.

Payne, read pages 409, Ex. 1-11, 19, 21, 23, 25, 27, 29, 35 pages 410-411.

Pearson, read pages 406-407 (examples 7 and 8), Ex. 46 a,c,e,f,h, j page 407.

Obj. 13

Nichols, read page 165, Ex. 1 all parts; 2 a,c,d,f,g,k,n,o; 3 a, c,f,h,i,l,m,n,q,r,u,w,x,z,a',c' pages 165-167.

Wollensak teaching tape C-3801 Open Phrase

C-3802 Open Sentence

SELF-EVALUATION 2

Obj. I. Complete the following proofs by writing the correct reason in the blank space provided.

8 1. Prove: $\forall_{x}\forall_{y\neq 0}\forall_{r\neq 0}\forall_{s\neq 0}\ \dfrac{\frac{x}{y}}{\frac{r}{s}} = \dfrac{x}{y}\cdot\dfrac{s}{r}$

Proof: Statements	Reasons
a. $\dfrac{\frac{x}{y}}{\frac{r}{s}} \quad \dfrac{\frac{x}{y}\cdot\frac{s}{r}}{\frac{r}{s}\cdot\frac{s}{r}}$	_____
b. $\dfrac{\frac{x}{y}\cdot\left(\frac{s}{r}\right)}{1}$	_____
c. $\dfrac{x}{y}\cdot\dfrac{s}{r}$	_____

10 2. Prove: $\forall_{x}\forall_{y\neq 0} -\left(\dfrac{x}{-y}\right) = \dfrac{x}{y}$

Proof: Statements	Reasons
a. $-\left(\frac{x}{-y}\right) = (-1)\left(\frac{x}{-y}\right)$	_____
b. $= \dfrac{(-1)\cdot x}{(-y)}$	_____
c. $= \dfrac{(-1)\cdot x}{(-1)\cdot y}$	_____
d. $= \dfrac{x}{y}$	_____

II. For each of the following expressions, write its equivalent in a single expression.

9 (3) $x \div \dfrac{2}{3}$ (4) $\dfrac{a}{b} \div \dfrac{1}{2b}$ (5) $\dfrac{-3}{4} \div \dfrac{m}{n}$

(6) $\dfrac{\frac{2}{3}}{\frac{5}{6}}$ (7) $\dfrac{\frac{-a}{b}}{\frac{c}{-d}}$ (8) $\dfrac{\frac{3}{2}}{\frac{x-y}{2}}$

11 (9) $-\dfrac{-5}{-3}$ (10) $-\dfrac{-5x}{-3x}$ (11) $-\dfrac{-w}{r}$

12 (12) $\dfrac{12}{\frac{6}{x}}$ (13) $\dfrac{\frac{x}{y}+1}{\frac{x}{y}-1}$ (14) $\dfrac{\frac{x+y}{x-y}}{\frac{x-y}{x+y}}$

(15) $\dfrac{\frac{a}{x} + \frac{a}{y}}{\frac{b}{x} - \frac{b}{y}}$ (16) $\dfrac{2 - \frac{1}{x}}{4 - \frac{1}{x}}$ (17) $\dfrac{x}{1 + \frac{1}{x}}$

13 III. Change the following word phrases to equivalent mathematical phrase.

18. the product of seven and the sum of some number and five.

19. two-hundred dollars less than one-third of last years salary.

20. number of feet in 18y inches.

21. number of inches in the perimeter of a square with x feet for the length of the side.

IV. True or False.

9 _____ 22. $\frac{3}{4} \div \frac{a}{b} = \frac{3a}{4b}$

_____ 23. $\dfrac{\frac{x}{y}}{\frac{a}{b}} = \frac{xb}{ya}$

_____ 24. $\frac{3}{5} \div \frac{x}{y} = \frac{5x}{3y}$

11 _____ 25. $\frac{(-x)3}{y(-7)} = -\frac{3x}{7y}$

_____ 26. $-\frac{1}{r - s} = \frac{1}{s - r}$

_____ 27. $-\frac{1}{x + y} = \frac{1}{-x - y}$

_____ 28. $-\frac{-3x}{-2y} = \frac{3x}{2y}$

12 _____ 29. $\dfrac{\frac{1}{2}}{\frac{2}{3}} = \frac{3}{4}$

_____ 30. $\dfrac{\frac{a - b}{a + b}}{\frac{a + b}{a - b}} = 1$

_____ 31. $\dfrac{\frac{a+b}{2}}{\frac{6}{a+b}} = \dfrac{1}{3}$

_____ 32. $\dfrac{\frac{x}{y} - \frac{3}{2}}{\frac{x}{y} + \frac{3}{2}} = \dfrac{2x - 3y}{2x + 3y}$

_____ 33. $\dfrac{1}{1 + \frac{1}{x}} = \dfrac{x}{x + 1}$

IF YOU HAVE SATISFACTORILY COMPLETED YOUR WORK, YOU MAY TAKE YOUR LAP TEST. CONSULT YOUR TEACHER FIRST.

ADVANCED STUDY

I. Complete the following charts:

LINEAR

Millimeter	Centimeter	Decimeter	Meter
1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
2	$\frac{2}{10}$		
3			
5			
x		$\frac{x}{100}$	
x + 2	$\frac{x + 2}{10}$		
10x		$\frac{x}{10}$	
10x + 10			

MASS		VOLUMETRIC	
Gram	Kilogram	(cc) Milliliter	Liter
1	$\frac{1}{1000}$	1	$\frac{1}{1000}$
2		7	
100	$\frac{1}{10}$	10	
x		x	
100x		1000x	
1000x + 1		0x + 2	

1. 400 millimeter, 20 centimeter is _____ meters.

2. x + 2 grams is _____ kilograms.

3. 10x milliliters is _____ liters.

II. Work Problems 1-16 page 325, Vanatta.

III.. Dolciani, p. 319, Just for Fun.

IV. Dolciani, pp. 328-330, Extra for Experts.

V. Work the following:

(1) $5x - \dfrac{3}{5x - \dfrac{3}{5x}}$

(2) $\left[\dfrac{2x + 1}{x} - \dfrac{x}{2x + 1}\right] \cdot \left[\dfrac{5x - 1}{x} + \dfrac{x}{5x - 1}\right]$

(3) $\left[\dfrac{4}{x + 1} + \dfrac{3}{x - 2}\right] \cdot \dfrac{x + 3}{7x - 5}$

(4) $\dfrac{2 + \dfrac{5a}{a + 2b}}{\dfrac{3a}{a + 2b}}$

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Wollensak teaching tape C-3801 - Open Phrase .
C-3802 - Open Sentence.